

Math 347

Homework 2 Solutions

2.4 #1 (a). We will model this problem as an unconstrained multivariable optimization problem. See the Maple file whale.mw for all calculations. Let T denote the total population growth rate (whales/year):

$$T = 0.05x\left(1 - \frac{x}{150000}\right) - 10^{-8}xy + 0.08y\left(1 - \frac{y}{400000}\right) - 10^{-8}xy,$$

where x is the blue whale population and y is the fin whale population. Our objective is to maximize T over the set of all (x, y) for which $x \geq 0$ and $y \geq 0$. We find that the maximum occurs at a population of 196,545 fin whales and 69,103 blue whales.

(b). Next we consider the sensitivity with respect to r_1 , the growth rate for blue whales. We compute $S(x, r_1) = 0.085$ and $S(y, r_1) = -0.0015$, so both optimal population levels are quite insensitive to the intrinsic growth rate for blue whales. If r_1 increases by 10%, then the optimal blue whale population increases by about 8.5%, and the optimal fin whale population decreases by about 0.015%. Thus the optimal blue whale population is more sensitive than the optimal fin whale population to the growth rate r_1 .

Next we consider the sensitivity with respect to r_2 , the growth rate for fin whales. We compute $S(x, r_2) = -0.0015$ and $S(y, r_2) = 0.018$, so both optimal population levels are quite insensitive to the intrinsic growth rate for fin whales. The optimal fin whale population is more sensitive than the optimal blue whale population to the growth rate r_2 .

(c). Next we consider the sensitivity with respect to K_1 , the carrying capacity for blue whales. We compute $S(x, K_1) = 1.001$ and $S(y, K_1) = -0.017$, so that if the carrying capacity for blue whales increases by 10%, then the optimal population for blue whales increases by about 10% and the optimal population for fin whales stays about the same.

Next we consider the sensitivity with respect to K_2 , the carrying capacity for fin whales. We compute $S(x, K_2) = -0.085$ and $S(y, K_2) = 1.001$, so that if the carrying capacity for fin whales increases by 10%, then the optimal population for fin whales increases by about 10% and the optimal population for blue whales stays about the same.

(d). Next we let α denote the competition parameter and consider whether or not it is ever possible for one species to become extinct. When $\alpha_1 = \alpha_2 > 1.25 \cdot 10^{-7}$, it is optimal to extinct the blue whales.

2.4 #3(a) Let P denote the total revenue obtained from harvesting (measured in \$1000/year). We have

$$P = 12\left(0.05x\left(1 - \frac{x}{150000}\right) - 10^{-8}xy\right) + 6\left(0.08y\left(1 - \frac{y}{400000}\right) - 10^{-8}xy\right),$$

where x is the blue whale population and y is the fin whale population. Our objective is to maximize P . We model this as an unconstrained multivariable optimization problem. See the Maple file whaleProfit.mw. The maximum revenue occurs at a population of 70,619 blue whales and 194,703 fin whales. At these population levels, the annual harvest is worth around \$68 million.

2.4 #6(a) We will use the following variables in the model:

p	price of computer (dollars / computer)
s	monthly sales (computers / month)
a	advertising budget (dollars / month)
C	manufacturing cost (dollars / month)
R	revenue (dollars / month)
P	profit (dollars / month)

We have:

$$\begin{aligned} C &= 700s + a \\ s &= 10000 + \frac{5000(950-p)}{100} + \frac{200(a-50000)}{10000} \\ R &= ps \\ P &= R - C \end{aligned}$$

Thus the profit P can be written as a function of the price and advertising budget: $P(p, a) = p\left(10000 + \frac{5000(950-p)}{100} + \frac{200(a-50000)}{10000}\right) - \left(700\left(10000 + \frac{5000(950-p)}{100} + \frac{200(a-50000)}{10000}\right) + a\right)$. Our goal is to maximize P over the constraint set $p \geq 0$ and $0 \leq a \leq 100000$. First, we look for possible maxima in the interior of the feasible region. See the Maple file advertise.mw. The only point at which the gradient is equal to zero is outside the feasible region, so there are no local extrema in the interior. Thus, we look along the constraint line $g(p, a) = a = 100000$. Note that the gradient of g is $\nabla g = [0, 1]$. Thus we solve the Lagrange multiplier equation $\nabla f = \lambda \nabla g$, which is equivalent to the system:

$$\begin{aligned} \frac{\partial P}{\partial p} &= 0 \\ \frac{\partial P}{\partial a} &= \lambda, \end{aligned}$$

along with the constraint equation $a = 100000$.

The maximum occurs when $p = 935$ and $a = 100000$. At this point, the profit is \$2,661,250.

On the line $g(p, a) = a = 0$, we find that the maximum occurs at $p = 915$ and $a = 0$, and the corresponding maximum profit is \$2,311,250. Finally, on the line $g(p, a) = p = 0$, the maximum occurs at the origin $(0, 0)$, and the corresponding profit is \$0. Thus the maximum over the entire feasible region is at the point $p = 935$ and $a = 100000$.