Math 333
Quiz 8
Thursday, April 10, 2008

Solutions

1. Let $p > 0$ and $q > 0$ denote real, positive constants, and let $g(t)$ be a continuous function of t. Suppose that $2e^{-t} + \sin t + 4$ is a *particular* solution of the differential equation $y'' + py' + qy = g(t)$. Let $y(t)$ denot the *general* solution of the differential equation. Describe the behavior of $y(t)$ as $t \to \infty$. Does the behavior of $y(t)$ depend on the initial conditions?

since $p, q > 0$, $y_h(t) \to 0$ as $t \to \infty$.

Thus $y(t) \to 2e^{-t} + \sin t + 4$ as $t \to \infty$.

2. Find the general solution of the differential equation $y''' - y'' - y' + y = 0$.

$r^3 - r^2 - r + 1 = 0 \quad r = -1, 1, 1$

$\Rightarrow \boxed{y(t) = k_1 e^{-t} + k_2 e^{t} + k_3 t e^{t}}$

3. Find the general solution of the differential equation $y''' - y' = 2\sin t$.

$r^3 - r = 0 \quad r(r^2-1) = 0 \quad r = 0, \pm 1$

$y_h(t) = K_1 + K_2 e^{-t} + K_3 e^t$

Guess $y_p(t) = A\sin t + B\cos t$

$y_p''' - y_p' = -2A\cos t + 2B\sin t = 2\sin t$

$2B = 2 \Rightarrow B = 1$

$-2A = 0 \Rightarrow A = 0$

$\Rightarrow y_p(t) = \cos t.$

The general solution is:

$$\boxed{y(t) = K_1 + K_2 e^{-t} + K_3 e^t + \cos t.}$$