Math 333 Quiz 6 Solutions

1. $6y'' - y' - y = 0$

$6r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{1 - 4 \cdot 6 \cdot -1}}{12} = \frac{1 \pm \sqrt{25}}{12} = \frac{1 \pm 5}{12} = \frac{1}{2}, -\frac{1}{3}$

$$\boxed{y(t) = K_1 e^{t/2} + K_2 e^{-t/3}}$$

2. $y'' + 2y' - 8y = 0$

$r^2 + 2r - 8 = 0 \quad (r+4)(r-2) = 0 \quad r = -4, 2$

$$\boxed{y(t) = K_1 e^{2t} + K_2 e^{-4t}}$$

3. $y'' - 2y' + y = 0$

$r^2 - 2r + 1 = 0 \quad (r-1)^2 = 0 \quad r = 1$

$$\boxed{y(t) = K_1 e^t + K_2 t e^t}$$

4. $t^2y'' + 2ty' - 2y = 0, t > 0$

(a) $y_1(t) = t \quad y_1' = 1 \quad y_1'' = 0$

$$t^2y_1'' + 2ty_1' - 2y_1 = t^2 \cdot 0 + 2t \cdot 1 - 2t$$
$$= 2t - 2t$$
$$= 0 \checkmark$$

(b) Use the reduction of order technique.

$$y(t) = v(t)y_1(t) = vt$$

$$y' = v + v't$$

$$y'' = v' + v''t + v' = 2v' + v''t$$

$$t^2y'' + 2ty' - 2y = 0$$

$$\Rightarrow t^2(2v' + v''t) + 2t(v + v't) - 2vt = 0$$

$$2t^2v' + t^3v'' + 2tv + 2t^2v' - 2tv = 0$$

$$4t^2v' + t^3v'' = 0$$

$$t^3v'' = -4t^2v'$$

$$v'' = -\frac{4}{t}v' \Rightarrow \int \frac{dv'}{v'} = \int -\frac{4}{t}\,dt$$

$$\ln|v'| = \ln t^{-4} + C$$

$$v' = At^{-4}$$

$$v(t) = K_1 t^{-3} + K_2$$

so the general solution is:

$$y(t) = v(t) \cdot t$$

$$= (K_1 t^{-3} + K_2)t$$

$$= K_1 t^{-2} + K_2 t$$

$$\boxed{y(t) = K_1 t^{-2} + K_2 t}$$