Math 333

Quiz 5

solutions

1. $r^2+2r-3=0$

$(r+3)(r-1)=0$

$r_1=-3,\ r_2=1$

$$\boxed{y(t)=k_1e^{-3t}+k_2e^{t}}$$

2. $r^2-(2\alpha-1)r+\alpha(\alpha-1)=0$

$$r=\frac{2\alpha-1\pm\sqrt{(2\alpha-1)^2-4\alpha(\alpha-1)}}{2}$$

$$=\frac{2\alpha-1\pm\sqrt{4\alpha^2-4\alpha+1-4\alpha^2+4\alpha}}{2}$$

$$=\frac{2\alpha-1\pm\sqrt{1}}{2}=\frac{2\alpha-1\pm1}{2}\Rightarrow r_1=\alpha,\ r_2=\alpha-1$$

So the general solution is:

$$y(t) = k_1 e^{\alpha t} + k_2 e^{(\alpha-1)t}$$

All solutions tend to 0 as $t \to \infty$ if

$\alpha < 0$ and $\alpha - 1 < 0$, i.e.

$$\boxed{\alpha < 0}$$

3. $$y'' + \frac{3t\,y'}{t(t-4)} + \frac{4y}{t(t-4)} = 0, \quad y(3) = 0, \quad y'(3) = -1$$

$\boxed{(0,4)}$ is the longest interval containing $t_0 = 3$ on which a unique solution is guaranteed to exist.

$\rightarrow$

4. First, show that y_1 and y_2 are solutions.

$$y_1 = x \quad y_1' = 1 \quad y_1'' = 0$$

$$\left(1 - x\frac{\cos x}{\sin x}\right) y_1'' - x \cdot y_1' + y_1 = 0 - x \cdot 1 + x = 0 \checkmark$$

$$y_2 = \sin x \quad y_2' = \cos x \quad y_2'' = -\sin x$$

$$\left(1 - x\frac{\cos x}{\sin x}\right) \cdot y_2'' - x y_2' + y_2 = \left(1 - x\frac{\cos x}{\sin x}\right) \cdot (-\sin x) - x \cdot \cos x + \sin x$$

$$= -\sin x + x\cos x - x\cos x + \sin x = 0 \checkmark$$

To show that y_1 and y_2 form a fundamental set of solutions, we show that $W(y_1, y_2) \neq 0$.

$$W(y_1, y_2) = y_1 y_2' - y_1' y_2$$

$$= x \cdot \cos x - 1 \cdot \sin x \neq 0$$

Thus, y_1 and y_2 form a fundamental set of solutions, so the general solution is:

$$\boxed{y(x) = K_1 x + K_2 \sin x}$$