

Math 333

Quiz 4

solutions

Thursday, February 14, 2008

1. Find all equilibrium points of the system

$$\frac{dx}{dt} = x - y + x^2 - xy$$

$$\frac{dy}{dt} = -y + x^2.$$

$$dy/dt = 0 \Rightarrow y = x^2$$

$$dx/dt = 0 \Rightarrow x - x^2 + x^2 - x^3 = 0$$

$$x(1 - x^2) = 0$$

$$x = 0, \pm 1$$

so the equilibrium points are:

$$(0, 0)$$

$$(-1, 1)$$

$$(1, 1)$$

2. Convert the second-order differential equation

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = 0$$

into a system of first-order differential equations, where

$$\frac{dy}{dt} = v.$$

$$\frac{dy}{dt} = v$$

$$\frac{dv}{dt} + 7v + 10y = 0 \Rightarrow \frac{dv}{dt} = -7v - 10y$$

3. Bonus (5 points). Find the general solution of the system

$$\begin{aligned}\frac{dx}{dt} &= 2x + 3y \\ \frac{dy}{dt} &= -4y.\end{aligned}$$

$$\frac{dy}{dt} = -4y \Rightarrow \frac{1}{y} dy = -4 dt \Rightarrow y = K_1 e^{-4t}$$

$$\frac{dx}{dt} = 2x + 3K_1 e^{-4t}$$

Associated homogeneous:

$$\frac{dx}{dt} = 2x \Rightarrow x_h(t) = K_2 e^{2t}$$

Guess: $x_p(t) = \alpha e^{-4t}$

$$\frac{dx_p}{dt} = -4\alpha e^{-4t}$$

$$\Rightarrow -4\alpha e^{-4t} = 2\alpha e^{-4t} + 3K_1 e^{-4t}$$

$$-4\alpha = 2\alpha + 3K_1 \Rightarrow -6\alpha = 3K_1 \Rightarrow \alpha = -\frac{1}{2} K_1$$

$$\Rightarrow x_p(t) = -\frac{1}{2} K_1 e^{-4t}$$

$$\boxed{\begin{aligned}x(t) &= K_2 e^{2t} - \frac{1}{2} K_1 e^{-4t} \\ y(t) &= K_1 e^{-4t}\end{aligned}}$$

Check:

$$\frac{dx}{dt} = 2K_2 e^{2t} + 2K_1 e^{-4t}$$

$$2x + 3y = 2K_2 e^{2t} + 2K_1 e^{-4t}$$

$$\frac{dy}{dt} = -4K_1 e^{-4t} = -4y$$