Math 333 Solutions.
Quiz 3
Thursday, February 7, 2008

1. Locate the bifurcation values for

$$\frac{dy}{dt} = y^2 - ay + 1$$

and describe the bifurcation that occurs at each bifurcation value.

$$\frac{dy}{dt} = y^2 - ay + 1 = 0 \Rightarrow y = \frac{a \pm \sqrt{a^2 - 4}}{2}$$

The bifurcation values occur at $a = \pm 2$.

<u>Case 1</u>: $a^2 - 4 < 0 \Rightarrow$ no equilibrium points

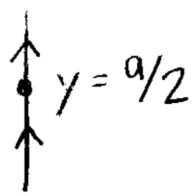
$a^2 < 4$

$-2 < a < 2$

<u>Case 2</u>: $a^2 - 4 = 0 \Rightarrow$ 1 equilibrium point, $y = a/2$.

$a^2 = 4$

$a = \pm 2$



<u>Case 3</u>: $a^2 - 4 > 0 \Rightarrow$ 2 equilibrium points, $y_1 = \frac{a + \sqrt{a^2-4}}{2a}$, $y_2 = \frac{a - \sqrt{a^2-4}}{2a}$

$a > 2$ or $a < -2$

2. Solve the initial-value problem

$$\frac{dy}{dt} + 2y = e^{t/3}, \quad y(0) = 1.$$

The general soln of the associated homogeneous DE is

$y_h(t) = ke^{-2t}$ Guess: $y_p(t) = \alpha e^{t/3}$

$\Rightarrow \frac{\alpha}{3} e^{t/3} + 2\alpha e^{t/3} = e^{t/3} \Rightarrow \frac{7}{3}\alpha = 1 \Rightarrow \alpha = \frac{3}{7}$.

$\Rightarrow y(t) = ke^{-2t} + \frac{3}{7} e^{t/3}$ $\quad y(0) = 1 \Rightarrow k = \frac{4}{7}$

$$\boxed{y(t) = \frac{4}{7} e^{-2t} + \frac{3}{7} e^{t/3}}$$

3. Find the general solution of

$$\frac{dy}{dt} = -\frac{y}{1+t} + t^2.$$

$\frac{dy}{dt} + \frac{1}{1+t} y = t^2$ $\quad g(t) = \frac{1}{1+t}$. $\quad \int g(t)\,dt = \ln(1+t)$.

$\mu(t) = e^{\ln(1+t)} = 1+t$

$$\Rightarrow y(t) = \frac{1}{1+t} \int (1+t) \cdot t^2\,dt = \boxed{\frac{1}{1+t}\left(\frac{1}{3}t^3 + \frac{1}{4}t^4 + C\right) = y(t)}$$