Math 333
Quiz 2
Thursday, January 31, 2008

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1. Sketch the phase line for the differential equation

$$\frac{dy}{dt} = -(y-1)(y-2)(y-3).$$

   Identify each equilibrium point as a sink, source, or node.

2. Find a polynomial function $f(y)$ of degree 4 such that the autonomous DE $\frac{dy}{dt} = f(y)$ has a node at $y = -1$, a sink at $y = 1$, a source at $y = 2$, and no other equilibrium points.

3. Consider the differential equation

$$\frac{dy}{dt} = -y^2.$$

(a) Show that

$$y_1(t) = \frac{1}{t-1} \text{ and } y_2(t) = \frac{1}{t-2}$$

are both solutions of the given differential equation.

(b) Let $y(t)$ be any solution of the differential equation with initial condition $y(0) = y_0$ that satisfies $-1 < y_0 < -\frac{1}{2}$. Why are we guaranteed that such a solution $y(t)$ always exists?

(c) Without explicitly solving the differential equation, find a lower and upper bound (in terms of t) for $y(t)$. Explain how you obtained these bounds, and if you apply any theorems, be sure that you explicitly state why those theorems apply to this problem.

(d) Describe the behavior of $y(t)$ as $t \to \infty$ and as $t \to -\infty$.