Math 333
Homework: Sinusoidal Forcing

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1. Consider the forced undamped oscillator described by the initial-value problem

$$y'' + y = 3\cos(\omega t), \quad y(0) = 0, \quad y'(0) = 0.$$

   (a) Find the solution of this initial-value problem. Note that you will need to consider the cases $\omega = 1$ and $\omega \neq 1$ separately. What is the natural frequency of the oscillator?

   (b) Plot the solution $y(t)$ vs. t for $\omega = 0.7$, $\omega = 0.8$, $\omega = 0.9$, and $\omega = 1$. Describe how the behavior of $y(t)$ changes as ω varies. What happens as ω takes on values closer and closer to 1? What happens when ω is exactly equal to 1?

2. Consider the modified oscillator system

$$y'' + \frac{1}{5}y' + y + \frac{1}{5}y^3 = \cos(\omega t), \quad y(0) = 0, \quad y'(0) = 0.$$

   This is called a spring-mass system with a hardening spring.

   (a) Graphically simulate solutions of the initial-value problem for several values of ω between 1/2 and 2, and estimate the amplitude of the steady-state response in each case.

   (b) Based on your data from part (a), for what value of ω is the amplitude greatest?

   (c) Compare the results of parts (a) and (b) with the corresponding results for the ordinary linear oscillator system $y'' + \frac{1}{5}y' + y = \cos(\omega t)$.