Math 333
Homework 4 Solutions

Part 1

2.1 #1: System (ii) corresponds to the system of large prey and small predators and system (i) corresponds to the system of small prey and large predators.

2.1 #2: For (i), the equilibrium points are $(0,0)$ and $(10,0)$. For (ii), the equilibrium points are $(0,0)$, $(0,15)$ and $(3/5,30)$.

2.1 #7(a): The population stars with a relatively large rabbit (R) population and a relatively small fox (F) population. The rabbit population grows, then the fox population rows while the rabbit population decreases. Next the fox population decreases until both populations are close to zero. Then the rabbit population grows again and the cycle repeats. Both populations oscillate toward an equilibrium which is approximately $(R,F)=(1/2,3/2)$.

2.1 #9: Only the equation for dR/dt needs modification. (i) $dR/dt = 2R - 1.2RF - \alpha$. (ii) $dR/dt = R(2-R) - 1.2RF - \alpha$

2.1 #10: $dF/dt = -F + 0.9RF - kF$ in each case

2.1 #11: For both (i) and (ii), $dR/dt = kF + 0.9RF$

2.1 #12: $dF/dt = kF(1 - F/N) + 0.9RF$

2.1 #13: One possibility is $dF/dt = -F + 0.9RF + k(R - 5F)$. There are many other possibilities as well.

2.1 #14: (i) $dR/dt = 2R - 1.2RF - kF$. (ii) $dR/dt = 2R - R^2 - 1.2RF - kF$.

2.1 #17:
(a) For the initial condition close to zero, the pest population increases much more rapidly than the predator. After a sufficient increase in the predator population, the pest population starts to decrease while the predator population keeps increasing. After a sufficient decrease in the pest population, the predator population starts to decrease. Then, the population comes back to the initial point.

(b) After applying the pest control, you may see the increase of the pest population due to the absence of the predator. So, in the short run, this sort of pesticide can cause an explosion in the pest population.

2.2 #10: The function $x(t)$ initially increases, reaches a maximum value, and then tends to zero as $t \to \infty$. It remains positive for all positive values of t. The function $y(t)$ decreases toward zero as $t \to \infty$.

2.2 #11: (a) (vii)

(b) (viii)

(c) (v)

(d) (vi)

2.2 #13: The equilibrium point is $(-1/9, 2/9)$.

2.2 #16: The equilibrium points are $(0,0)$, $(1,0)$, and $(-1,0)$.

2.2 #17: The equilibrium points are $(\pi/2 + k\pi, 0)$ for any integer k.

Part 2

2.1 #19(a):

$$\begin{aligned}\frac{d^2y}{dt^2} + y &= \frac{d^2(\sin t)}{dt^2} + \sin t \\ &= -\sin t + \sin t \\ &= 0.\end{aligned}$$

2.1 #19(b): Phase plot.

2.1 #20(a): $\beta = \sqrt{\frac{k}{m}}$

2.2 #7: $dy/dt = v$, $dv/dt = y$

2.2 #8: $dy/dt = v$, $dv/dt = -2y$

2.2 #19: $dx/dt = v$, $dv/dt = 3x - x^3 - 2v$. The equilibria are $(\pm\sqrt{3}, 0)$.