Math 333
Homework 3 Solutions

Section 1.7

1.7 #2: The bifurcation value is $a = 9/4$.

1.7 #4: The bifurcation value is $\alpha = 0$.

1.7 #8: For $\alpha < -1$, solutions are always decreasing. For $\alpha = -1$, the equilibrium point $y = 0$ is a node. If $1 < \alpha < 0$, then there are two equilibrium points $y = \pm\sqrt{\ln(-1/\alpha)}$. If $\alpha \geq 0$, solutions are always increasing.

1.7 #10: The bifurcation value is $\alpha = -1$.

1.7 #13: No such $f(y)$ is possible.

1.7 #14: No such $g(y)$ is possible.

1.7 #17: (a) A model for the fish population that includes fishing is

$$\frac{dP}{dt} = 2P - \frac{P^2}{50} - 3L,$$

where L is the number of licenses issued. We want to find L as large as possible so that there is still an equilibrium point present. This is the bifurcation value of L, which occurs at $L = 50/3$. Since this value of L is not an integer, the largest number of licenses that should be allowed is 16.

(b) Slope field or graph. If we sell 16 licenses, then any initial population grater than 40 tends to the equilibrium level $P = 60$.

(c) The maximum number of licenses is $16\frac{2}{3}$. However, it is dangerous to allow this many licenses since the death of a few extra fish would push the number of fish below the equilibrium value $P = 50$, which would cause the fish population to die out.

Section 1.8

1.8 #2: $y(t) = ke^{-4t} + e^{-t}$

1.8 #4: $y(t) = ke^{2t} - \frac{1}{4}\cos(2t) - \frac{1}{4}\sin(2t)$

1.8 #6: $y(t) = ke^{t/2} + 4te^{t/2}$

1.8 #8: $y(t) = \frac{43}{4}e^{2t} - \frac{3}{4}e^{-2t}$

1.8 #19: Let $y(t) = y_h(t) + y_1(t) + y_2(t)$. Then $\frac{dy}{dt} + a(t)y = 0 + b_1(t) + b_2(t)$, so $y(t)$ is a solution of the original DE.

1.8 #20: $y_p(t) = \frac{3}{2}t^2 - \frac{1}{2}t - \frac{1}{4}$

1.8 #21: Use the technique suggested in Exercise 19. Calculate two solutions, one for the right hand side $t^2 + 2t + 1$ and one for the right hand side e^{4t}. Then the solution of the IVP is $y(t) = \frac{-5}{12}e^{-2t} + \frac{1}{2}t^2 + \frac{1}{2}t + \frac{1}{4} + \frac{1}{6}e^{4t}$.

1.8 #33(b): $\frac{d(y_p - y_q)}{dt} = a(t)(y_p - y_q)$.

Section 1.8

1.9 #2: $y(t) = \frac{t^6}{3} + ct^3$

1.9 #6: $y(t) = t^2(t-1)e^t + ct^2$

1.9 #10: $y(t) = 4t4^{-t^2} + 3e^{-t^2}$

1.9 #14: $y(t) = 4e^{t^3/3} \int e^{-t^3/3}\, dt$

1.9 #16: $y(t) = 4e^{-1/t} \int e^{1/t} \cos t\, dt$

1.9 #20: The values of r that give solutions in terms of elementary functions are $r = 0$ and $r = -1$.