A Systematic Technique for Optimal Bicycle Wheel Selection

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Introduction

We present a theoretical investigation of the dynamics and aerodynamics of a racing bicycle. We identify the dominant physical mechanisms and apply Newtonian mechanics principles to obtain a differential equation that gives the power output required of the rider. We then approximate the time-averaged power difference between the two types of rear wheel. We develop an easy-to-read, unambiguous, and comprehensive table that allows a person familiar with track and wind conditions to select the correct wheel type. We apply this table in the analysis of a time-trial stage of the Tour de France. We compare and contrast our choice of wheel with that of the leading competitors, with enlightening results.

Our criterion on the wind speed and direction gives predictions that agree with the exhaustive experimental data considered. We complement our criterion with suggestions based on the experience of an extensive range of experienced cyclists. We provide an informative critique of our model and suggest innovative ways to enhance the wheel-choice criterion.

Assumptions

- The spoked wheel is the Campagnolo Vento 16 HPW clincher wheel, used by the ONCE cycling team [ONCE Cycling Team Website 2000]. Reported mass = 1.193 kg [Hi-Tech Bikes 2001].

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Table 1.
Model Inputs and Symbols.

s	distance over which the average power is calculated
k	road grade
u	initial speed of the rider before any incline
c_r	coefficient of air drag for the bike and the rider frame
A_r	cross-sectional area of bike and rider frame
ρ	air density
$m(spk)$	mass of spoked rear wheel
$m(sld)$	mass of disc rear wheel
c_r	coefficient of rolling resistance
g	$9.81\mathrm{m/s}^2$ acceleration due to gravity
c_{fw}	drag coefficient of front wheel
$c_{rw}(sld)$	drag coefficient of solid wheel
$c_{rw}(spk)$	drag coefficient of spoked wheel
r_{rw}	radius of rear wheel
μ	constant of deceleration
ϕ	angle of wind with respect to direction of bike (degrees)
v_f	speed of the wind with respect to the cyclist
I_{rw}	moment of inertia of the rear wheel
I_{fw}	moment of inertia of the front wheel
v_w	wind speed
ν	kinematic viscosity of air

- The density and pressure of the air across the bike are essentially constant. (See **Appendix**. [EDITOR'S NOTE: We omit the **Appendix**.])
- For most of the journey, the bike travels in a straight line ($\pm 10\%$). This is reasonable, since cyclists avoid turning as much as possible and they have a wide enough road to achieve a straight line.
- The solid wheel is taken to be the HED disc tubular freewheel, the solid wheel of choice in the Tour de France. Reported mass = 1.229 kg [Hi-Tech Bikes 2001].
- The drag coefficients of both wheels are independent of the wind speed in the range of raceable weather conditions and vary significantly with the relative direction of the wind [Flanagan 1996].
- The drag area of a typical crouched racer is 0.3 m^2 [Compton 1998].
- The radius of the wheel is 35 cm [Compton 1998].
- The efficiency of the drive train is essentially 100%, reasonable for élite racing bikes [Pivit 2001].
- The coefficient of rolling resistance between road and wheel rubber is 0.007 [Privit 2001].
- The moment of inertia of a bicycle wheel about an axis through its centre and perpendicular to the plane of the wheel is $I = \frac{1}{9}mr^2$, where m is the mass

of the wheel and r is the radius. This agrees with empirical data [Compton 1998].

- The deceleration to terminal speed on a uniform incline is constant to within $\pm 2\%$.
- The terminal speed on a uniform incline is reached at 100 m up the incline.
- The deceleration of a rider on a slope is proportional to the gradient of the slope (given in the problem statement).
- The power is averaged over 100 m of acceleration, where the acceleration is that calculated in the **Appendix**. This power is used in determining the wind speed criterion for solid wheels.
- All of the drag due to the rider and bike frame is due to the rider cross-sectional area, since the area of the rider is much greater.
- The rolling resistance $c_{rr}mg$ is the same for the solid wheel as for the spoked wheel. The tires surrounding the wheels are identical and the mass difference between the wheels is only about 0.1% of the overall mass of the bike plus rider.

The Wind Speed Criterion

We have from Newton's Second Law that $F = ma$, where F is the force acting the object, m is its mass, and a is its net acceleration. For the bicycle, the force equation can be written as follows:

$$\text{Force applied by rider} = \text{Retarding Forces} + ma + \text{rotational acceleration},$$

where m is the mass of the rider-bike system. The retarding forces consist of

- the drag due to the bike frame and the rider,
- the drag due to the individual wheels,
- the friction due to the motion of the wheels in the air, and
- the rolling resistance of the wheels on the surface.

The formulas for these torques and forces are

$$\begin{aligned}
\text{drag force of bike frame and rider} &= \frac{c_w A\rho v_f^2}{2}, \\
\text{drag force due to front wheel} &= \frac{c_{fw} A\rho v_f^2}{2}, \\
\text{drag force due to rear wheel} &= \frac{c_{rw} A\rho v_f^2}{2}, \\
\text{frictional torque} &= 0.616\pi\rho\omega^{3/2}\nu^{1/2}r^4, \\
\left(m_r + m_b + m_{fw} + \frac{I_{fw}}{r^2} + m_{rw} + \frac{I_{rw}}{r^2}\right)a &= \text{Resultant Force.}
\end{aligned}$$

With this in mind, the power is averaged. The criterion on the wind speed v_w becomes:

$$v_w > \sqrt{\frac{m_{rw}(sld) - m_{rw}(spk)}{(\sigma_{spk} - \sigma_{sld})} + \left(\frac{\gamma}{s}\right)^2 - \frac{\lambda}{s}} + \frac{\gamma}{s},$$

where σ, λ, and γ are as given in the **Appendix**. If the quantity on the left side is not positive, then the solid wheel is always best. The quantity σ is effectively an aerodynamic term, whereas γ and λ can be thought of as representing the acceleration, and time of acceleration respectively. Note that γ is dependent on the wind direction ϕ.

The Minimum Wind Speed Table

Table 2 gives the wind speed below which the power for the spoked wheel is less, for various rider speeds. A 0 means that for *any nonzero* wind speed, the solid wheel requires less power; ∞ means that the rider will not be able to continue up the slope for 100 m.

Course Example

Time-trial course: Stage 1, Tour de France 2000 (**Figure 1**): A 16.5 km circuit starting at the Futuroscope building in Chassenuil-du-Poitou, France [Tour de France 2000].

Modeling the course: The course is modeled by a quadrilateral. This is a faithful representation of the course [CNN/Sports Illustrated Website 2000]. This course is predominantly level (**Figure 2**) save for a 1 km climb at a gradient of 3.7%. This climb is treated as the crucial feature of the time trial course, insofar as wheel choice is concerned.

Table 2.

Wind speed (in m/s) below which the power for the spoked wheel is less.

a.

Rider speed of 10 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	16	0	0	0	0
0.01	23	0	0	0	0
0.02	29	1	0	0	3
0.03	34	3	0	0	5
0.04	39	4	0	0	7
0.05	43	6	1	0	8
0.06	48	7	3	0	10
0.07	51	8	4	0	11
0.08	55	10	5	1	13
0.09	59	11	6	2	14
0.10	∞	∞	∞	3	∞

b.

Rider speed of 12.5 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	13	0	0	0	0
0.01	21	0	0	0	0
0.02	25	0	0	0	0
0.03	26	0	0	0	2
0.04	32	2	0	0	3
0.05	36	3	0	0	5
0.06	41	4	0	1	7
0.07	45	5	0	2	8
0.08	48	7	1	3	10
0.09	52	8	2	4	11
0.10	59	9	3	5	12

c.

Rider speed of 14 m/s.

Wind direction	180°	173°	165°	150°	135°
Road Grade					
0.00	12	0	0	0	0
0.01	19	0	0	0	0
0.02	25	0	0	0	0
0.03	30	0	0	0	0
0.04	35	0	0	0	1
0.05	39	1	0	0	3
0.06	43	3	0	0	5
0.07	47	4	0	0	6
0.08	50	5	0	1	8
0.09	54	6	0	2	9
0.10	57	7	1	3	10

Weather: The air temperature was about 20°C. The wind was variable west to southwest, between 20 and 30 kph (5.6–8.3 m/s) [Official Tour de France Website 2000].

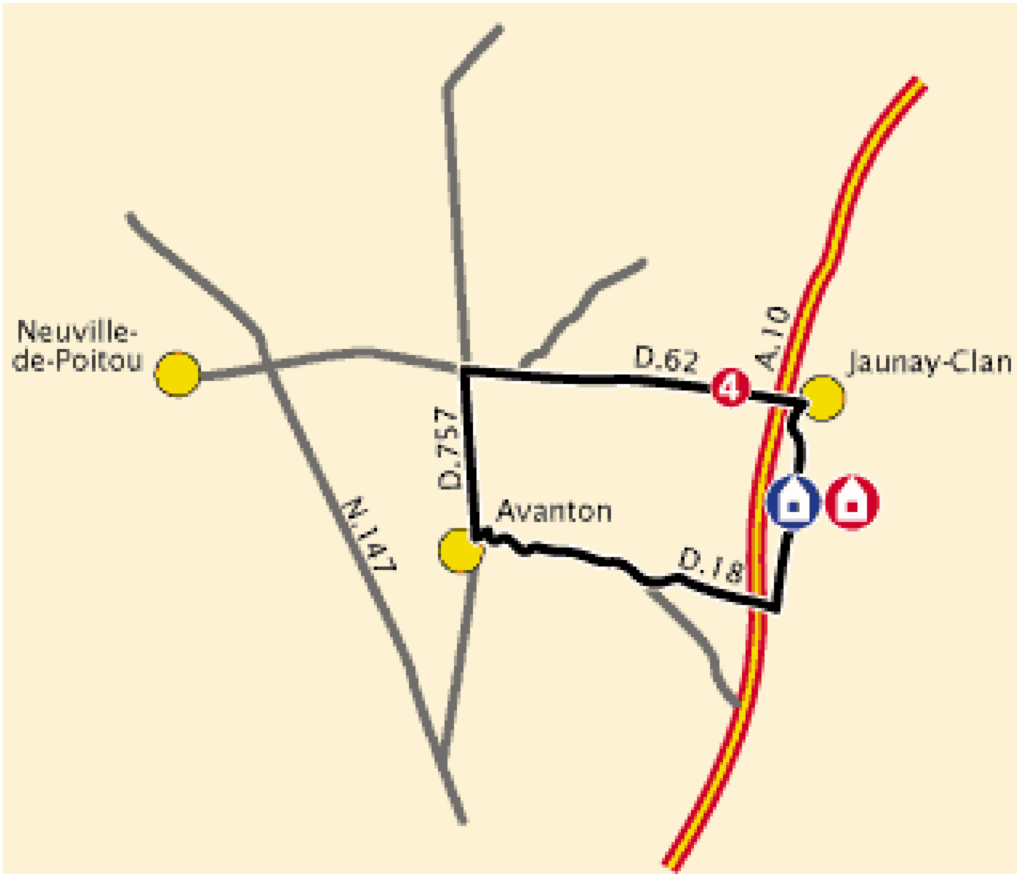

Figure 1. The course.

North points towards the top of the page. The climb is westward, meaning that the riders faced the wind at an angle varying from 0° to 45° to the wind: exactly the range of applicability of the table. The average speed of the top ten finishers was about 14 m/s. We analyse the climb from the perspective of a rider who begins the climb at that speed and whose speed levels off after 100 m of the climb. Knowing the speed of the rider to be 14 m/s and the gradient of the slope to be approximately 4%, we can apply **Table 2c**. The minimum wind speed in the table is 1.44 m/s (assuming that the wind is at an angle to the direction of motion). On the basis of the climb alone, the solid wheel is the better choice. In fact, assuming that the wind is at an angle to the rider for most of the journey (wind that "follows" a cyclist around is unlikely), the solid wheel is better at *all* nonzero wind speeds (assuming level terrain and rider speed of 14 m/s).

The recommended wheel for this race is therefore the solid wheel. In this time trial, U.S. Postal Team and the Spanish ONCE Team used a solid rear wheel and a tri-spoke front wheel (a more aerodynamically efficient version

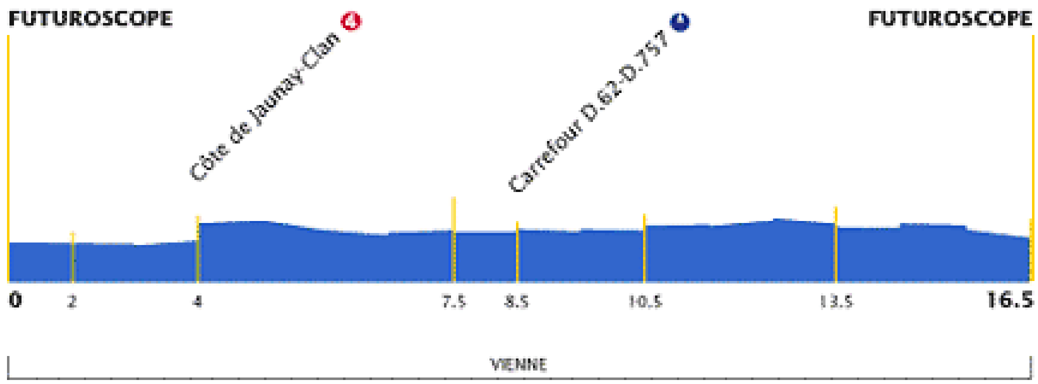

Figure 2. Profile of the course.

of the standard spoked wheel). The two teams had five riders in the top ten [Official Tour de France Website 2000; Pediana 2000].

The Adequacy of the Table

Comparing Its Predictions with Experimental Data

On an indoor circuit, the gradient and the wind can be considered to be zero. The corresponding entries in **Table 2**, for a direct headwind (180°) at zero gradient, are nonzero, indicating that spoked wheels are better. This prediction is borne out by the empirical results on an indoor circuit [Beer 1999] (**Table 3**).

Table 3.
Measurements made on a 1-km indoor circuit.

Watts		Spoked Rear Wheel	Solid Rear Wheel
100	time (min:sec)	2:22.58	2:26.78
	speed (ms^{-1})	6.97	6.77
200	time (min:sec)	1:51.75	1:52.55
	speed (ms^{-1})	8.90	8.83
300	time (min:sec)	1:36.98	1:39.70
	speed (ms^{-1})	10.25	9.97

The times correspond to seven laps of 147 m. The test cyclist rode at power outputs of 100 W, 200 W, and 300 W. The tests were repeated with no more than a 2% time variation; the spoked rear wheel has the better performance by a margin greater than the experimental error.

However, for a road circuit (where one would expect variable and nonzero wind velocities), **Table 2** predicts that for nonzero cross winds and slopes of a gradient less than 5%, the solid wheel is best, based on minimising aerodynamic losses. Thus, if the solid wheel is the right choice for a particular cyclist speed, then the solid wheel choice is valid for all greater cyclist speeds, given the same atmospheric conditions. Experimental data in **Table 4** back up this claim.

Table 4.
Measurements made on a 7.2-km road circuit.

Watts		Spoked Rear Wheel	Solid Rear Wheel
200	time (min:sec)	15:22	13:53
	speed (ms^{-1})	7.81	8.63

The circuit consisted of a 7.2-km looped course of rolling hills (gradient $< 5\%$) with varying wind conditions (nonzero wind speeds, unlike the indoor track). The power output of the cyclist was approximately 200 W. The tests were repeated with no more than a 2.3% time variation. For the road circuit, the solid rear wheel is the better choice (again by more than the experimental error). According to the appropriate table, the solid rear wheel is the more efficient for most of the conditions that were encountered. Thus, the predictions of the model are once more confirmed.

Additional Factors Not Considered by the Table

Stability of the bike: Stability is a major factor in cycling. A cyclist wants to concentrate on putting the maximum possible power through the pedals. If the bicycle is unstable, the jerking of the handlebars in response to sudden gusts of wind will be very disruptive. In general, bicycles with standard spoked wheels front and rear are the least affected by changing crosswinds, but bikes with solid wheels are fastest. A solid front wheel can increase the rider's pedaling wattage by 20% to 30%, but it removes all the bicycle's self-centering effects, making for a difficult ride. A solid rear wheel has similar but milder effects [Cobb 2000].

Turning radius: This factor is important if the course contains many turns or if maneuvering in response to other riders is necessary. A greater turning radius loses time at each turn, where more maneuverable riders may overtake.

Rider comfort: The solid wheel also creates the problem of rider comfort. As it has no spokes, it cannot flex to absorb any shocks due to irregularities of the road surface. Towards the end of a long race, a rider's concentration may be impaired due to the resulting discomfort, causing a drop in performance [Bicycle Encyclopedia 2000].

Very few cyclists ride with two solid wheels (1 out of 177 in the time trial that we studied), despite the aerodynamic superiority—steering problems negate the gain. Similarly, we have shown the superiority of the solid rear wheel in race conditions, yet in the long-distance (150+ km) stages of the Tour de France, all riders use spoked wheels: Solid wheels have insufficient maneuverability and steering problems cause mental fatigue near the end of the stage.

Thus, the *directeur sportif* should consider the length of the race, the wind conditions, and the quality of the road surface in making the decision.

Error Analysis

The error was determined by calculating the differential.

The wind direction may vary considerably over a given course due to local effects (e.g., buildings, trees). We chose an error of 10% for $c_{fw}(\phi)$ and $c_{rw}(\phi)$.

Air density can usually be determined to a high accuracy but varies by location. Therefore, we chose a 1% error, since the necessary equipment is unlikely to be available to the *directeur sportif* at the time of the race.

Using a wind tunnel, drag coefficients can be determined to a very high precision (to within 0.02kg); but variables such as pedaling speed, wheel rotation, and rider posture introduce inaccuracies that cannot be easily determined. The main difficulty is that the flowfield about a rider in a wind tunnel is not ergodic, primarily as a result of airflow irregularities caused by pedaling. Therefore, we chose an error of 3% for c_r [Flanagan 1996].

For each rider/bicycle combination, the *directeur sportif* should determine the appropriate masses and dimensions. We assume that they can be determined to a high degree of accuracy ($\sim 0.1\%$) with the exception of the rider's cross section, in which we take an error of 2%. From error analysis in the **Appendix**, we find

$$\frac{\triangle v_w}{v_w} = 16\%.$$

Strengths and Weaknesses

Our model clearly and concisely states which wheel should be used under the various conditions. The model is based on equations that are sufficiently versatile so that further factors pertaining to a particular situation may be included as required, without any need to construct new equations.

We could not verify the model experimentally under conditions of the high power output of élite cyclists. However, in data available for various lesser power outputs, we could not discern any noticeable trends in time differential between the different wheels with respect to increasing power.

Our model does not contain a quantitative analysis of the wind speeds at which the solid wheel is unacceptably unstable. However, this question depends largely on the abilities and preferences of each individual rider and requires detailed local information about road conditions and wind variability.

No data were available regarding the coefficients of drag in a tail wind. However, since the cyclist (~ 15 m/s) in general travels faster than the wind (race conditions $< 10\,\mathrm{m/s}$), only the crosswind effects are important, and these are adequately handled in the tables.

The major weakness of the model is assuming that the rider's speed does not differ much from the rider's average speed over the duration of the race. Moreover, we could find no evidence to support the problem's assumption that a rider reaches terminal velocity after 100 m of a slope.

Conclusion

- When there is a head wind, the spoked wheel is better. If the course is flat, the solid wheel is better for strong winds; however, the wind (particularly if gusting) may cause instability problems.
- When the wind is not weak (> 5 m/s) and strikes the wheel at an angle, the solid wheel is nearly always better. Even a small component of wind perpendicular to the rider direction makes the solid wheel the better choice.
- If the circuit has many tight turns or involves riding in close company with other cyclists, the solid wheel's lack of maneuverability dictates the spoked wheel; otherwise, the risk of an accident and injury is unacceptable.
- The region of superiority of the solid wheel increases with rider speed. Since the power required to overcome air resistance goes as the cube of the velocity, the aerodynamic savings of the solid wheel become more important with higher speed.

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Dr. Ann Watkins, President of the Mathematical Association of America, congratulating MAA Winners Eamonn Long, Michael Flynn, and William Whelan-Curtin, after they presented their model at MathFest in Madison, WI, in August. [Photo courtesy of Ruth Favro.]