Math 112 Solutions.
Quiz 7
Wednesday, April 9, 2008

1. Find the sum of the series $\sum_{j=2}^{\infty} \frac{3^j}{4^{j+1}}$. No simplification is necessary.

$$\sum_{j=2}^{\infty} \frac{3^j}{4^{j+1}} = \frac{3^2}{4^3} + \frac{3^3}{4^4} + \frac{3^4}{4^5} + \cdots$$ ← geometric with $a = 3^2/4^3$ and $r = 3/4$.

Since $|3/4| < 1$, the series converges to

$$\boxed{\frac{3^2/4^3}{1 - 3/4} = 9/16}$$

2. Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converges or diverges.

$\frac{n}{n^3+1} \leq \frac{n}{n^3} = \frac{1}{n^2}$. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges b/c $p = 2 > 1$.

Thus, $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converges by the Comparison Test.

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^3}{5^n}$ converges or diverges.

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^3}{5^{n+1}}}{\frac{n^3}{5^n}} \right| = \lim_{n\to\infty} \left| \left(\frac{n+1}{n}\right)^3 \cdot \frac{5^n}{5^{n+1}} \right| = \frac{1}{5} < 1,$$

so the series converges by the Ratio Test.

4. Determine whether the series $\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j^2}$ converges or diverges.

$\sum_{j=1}^{\infty} \left| \frac{(-1)^{j+1}}{j^2} \right| = \sum_{j=1}^{\infty} \frac{1}{j^2}$ converges since $p=2>1$.

Thus, $\sum_{j=1}^{\infty} \frac{(-1)^{j+1}}{j^2}$ converges absolutely and hence converges.

Alternatively, you can use the Alternating Series Test:

(i) $\frac{1}{(j+1)^2} \leq \frac{1}{j^2}$ and (ii) $\lim_{j\to\infty} \frac{1}{j^2} = 0$.