Math 112 Quiz ~~5~~ 6 Wednesday, April 2, 2008

$\boxed{\text{Solutions}}$

1. Find the limit of the sequence $\{a_k\}$ defined by $a_k = (1.1)^k$, or explain why the limit does not exist.

$\lim_{k \to \infty} (1.1)^k$ DNE since $|1.1| > 1$

2. Find the limit of the sequence $\{a_n\}$ defined by $a_n = n \sin\left(\frac{1}{n}\right)$, or explain why the limit does not exist.

$$\lim_{n \to \infty} n \sin\left(\frac{1}{n}\right) = \lim_{n \to \infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \overset{L}{=} \lim_{n \to \infty} \frac{\cos\left(\frac{1}{n}\right) \cdot -\frac{1}{n^2}}{-\frac{1}{n^2}}$$

$$= \lim_{n \to \infty} \cos\left(\frac{1}{n}\right) = \cos 0 = \boxed{1}$$

3. Determine whether the sequence $\{a_n\}$ defined by $a_n = \dfrac{\cos^2 n}{2^n}$ converges or diverges. If the sequence converges, find its limit.

$$0 \leq \frac{\cos^2 n}{2^n} \leq \frac{1}{2^n} \qquad \lim_{n\to\infty} 0 = 0 \qquad \lim_{n\to\infty} \frac{1}{2^n} = 0$$

By the squeeze Theorem,

$$\lim_{n\to\infty} \frac{\cos^2 n}{2^n} = 0.$$

4. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{2n-3}$ converges or diverges. If the series converges, find the sum.

$\displaystyle\lim_{n\to\infty} \frac{n+1}{2n-3} = \frac{1}{2} \neq 0$, so the series diverges

by the Test for Divergence

5. Determine whether the series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ converges or diverges. If the series converges, find the sum.

Using partial fractions, $\frac{2}{n^2-1} = \frac{1}{n-1} - \frac{1}{n+1}$.

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right)$$

$$S_n = 1 + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$$

$$\lim_{n \to \infty} S_n = \frac{3}{2}.$$

The series converges to $\frac{3}{2}$.