Math 112 Solutions
Quiz 5
Wednesday, March 19, 2008

1. Show that
$$y = \sqrt[3]{x+C},$$

where C is a constant, is a solution of the DE $y' = \dfrac{1}{3y^2}$.

$$y = (x+C)^{1/3} \quad y' = \frac{1}{3}(x+C)^{-2/3}$$

$$\frac{1}{3y^2} = \frac{1}{3(x+C)^{2/3}} = \frac{1}{3}(x+C)^{-2/3} \qquad \Rightarrow y' = \frac{1}{3y^2} \checkmark$$

2. Show that
$$y = x^{-1} - 1$$

is a solution of the DE
$$x^3 y'' + x^2 y' - xy = x.$$

$$y = x^{-1} - 1 \quad y' = -x^{-2} \quad y'' = 2x^{-3}$$

$$x^3 y'' + x^2 y' - xy = x^3 \cdot 2x^{-3} + x^2 \cdot -x^{-2} - x(x^{-1} - 1)$$

$$= 2 - 1 - 1 + x$$

$$= x \checkmark$$

3. Find the solution of the IVP $y' = x^2 y$, $y(0) = -2$.

$$\frac{dy}{dx} = x^2 y \qquad \int \frac{1}{y}\, dy = \int x^2\, dx \qquad \ln|y| = \frac{1}{3} x^3 + C$$

$$|y| = e^C e^{\frac{1}{3}x^3} \qquad y = Ae^{\frac{1}{3}x^3} \qquad -2 = Ae^0 = A$$

$$\boxed{y = -2e^{\frac{1}{3}x^3}}$$

4. Find the general solution of the DE

$$y' = 4 - y^2.$$

$$\frac{dy}{dx} = 4 - y^2 \qquad \frac{1}{4-y^2}\, dy = dx \qquad \frac{1}{(2-y)(2+y)} = dx$$

$$\frac{1}{(2-y)(2+y)} = \frac{A}{2-y} + \frac{B}{2+y} \qquad 1 = A(2+y) + B(2-y)$$
$$y = -2 \quad 1 = 4B \quad B = \frac{1}{4} \qquad y = 2 \quad A = \frac{1}{4}$$

$$\int \left(\frac{\frac{1}{4}}{2-y} + \frac{\frac{1}{4}}{2+y} \right) dy = \int dx \qquad -\frac{1}{4}\ln|2-y| + \frac{1}{4}\ln|2+y| = x + C$$

$$\ln|2+y| - \ln|2-y| = 4x + C$$

$$\boxed{\ln\left| \frac{2+y}{2-y} \right| = 4x + C}$$