Math 112
Quiz 4
Wednesday, February 13, 2008

Solutions

The Error Bound Theorem Formulas

- $|I - L_n| \leq \frac{K_1(b-a)^2}{2n}$ and $|I - R_n| \leq \frac{K_1(b-a)^2}{2n}$
- $|I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}$ and $|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2}$

1. Let $I = \int_0^1 \sin(x^2)\, dx$. Explain why the inequalities

$$L_7 \leq I \leq R_4$$

are valid.

$f(x) = \sin(x^2)$ is increasing on $[0,1]$, so for any integers m and n, $L_m \leq I \leq R_n$.

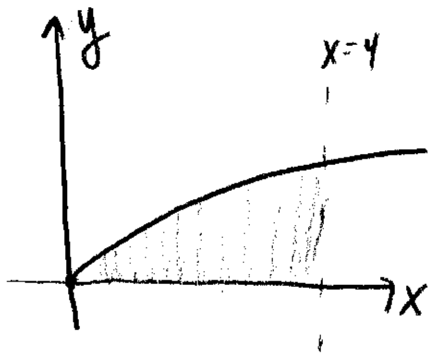
2. Find a value of n for which the Error Bound Theorem guarantees that L_n approximates the value of the integral within ± 0.005. Justify your answer. Maple is permitted for this problem (for arithmetic only).

$f'(x) = 2x\cos(x^2)$ $\quad |f'(x)| \leq 2$. Use $K_1 = 2$.

$$\Rightarrow |I - L_n| \leq \frac{2(1-0)^2}{2n} \leq 0.005$$

$$\Rightarrow n \geq \frac{2}{2 \cdot 0.005} = 200.$$

3. Find the area of the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$.



$$A = \int_0^4 \sqrt{x}\, dx = \frac{2}{3} x^{3/2} \Big|_0^4$$

$$= \frac{2}{3} \cdot 4^{3/2} = \boxed{\frac{16}{3}}$$

4. Find the length of the curve

$$y = \frac{x^2}{2} - \frac{\ln x}{4}$$

from $x = 3$ to $x = 9$.

$$y' = \frac{2x}{2} - \frac{1}{4x} = x - \frac{1}{4x}$$

$$L = \int_3^9 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx = \int_3^9 \sqrt{1 + x^2 - \frac{1}{2} + \frac{1}{16x^2}}\, dx$$

$$= \int_3^9 \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\, dx = \int_3^9 \sqrt{\left(x + \frac{1}{4x}\right)^2}\, dx = \int_3^9 \left(x + \frac{1}{4x}\right) dx$$

$$= \boxed{36 + \frac{1}{4} \ln 3}$$