Math 112
Quiz 3
Wednesday, February 1, 2008

Solutions

Some Useful Summation Formulas.

- $\sum_{k=1}^{n} c = cn$
- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

1. Express the sum

$$1^2 + 3^2 + 5^2 + \cdots + 97^2 + 99^2$$

using sigma notation.

$\sum_{i=0}^{49} (2i+1)^2$ or $\sum_{i=1}^{50} (2i-1)^2$

2. Evaluate $\sum_{k=0}^{9} (k+1)^2$. $= 1^2+2^2+3^2+\cdots+10^2$

$\sum_{k=1}^{10} k^2 = \frac{10(10+1)(20+1)}{6} = \frac{10 \cdot 11 \cdot 21}{6}$

3. Evaluate $\int_2^5 x\,dx$ by computing the limit of Riemann sums (either left or right sums are fine; use whichever you prefer).

$f(x) = x$

$\Delta x = \frac{5-2}{n} = \frac{3}{n}$

$$\int_2^5 x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(2 + \frac{3i}{n}\right) \cdot \frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2 + \frac{3i}{n}\right) \frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(2 + \frac{3i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(2n + \frac{3}{n} \cdot \frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty} \left(6 + \frac{9}{n^2} \cdot \frac{n^2+n}{2}\right)$$

$$= 6 + \frac{9}{2} = \boxed{\frac{21}{2}}$$