Exam 2 Practice

1. Consider the differential equation
$$\frac{dy}{dx} = \frac{x}{y}.$$
   (a) Find the general solution of the differential equation.
   (b) Find the particular solution that satisfies $y(2) = 1$.

2. Determine whether each of the following improper integrals converges or diverges.
   (a) $\displaystyle\int_3^\infty \frac{\ln x}{x}\,dx$
   (b) $\displaystyle\int_1^\infty \frac{x}{x^5+1}\,dx$
   (c) $\displaystyle\int_0^4 \frac{1}{x-3}\,dx$

3. Determine whether each of the following sequences $\{a_n\}$ converges. If the sequence converges, find its limit.
   (a) $a_n = \frac{\ln n}{n}$
   (b) $a_n = \left(\frac{1}{2}\right)^n$
   (c) $a_n = n$
   (d) $a_n = \left(1 + \frac{2}{n}\right)^n$

4. Find the sums of the following series.
   (a) $\displaystyle\sum_{n=1}^\infty \frac{1}{n(n+2)}$
   (b) $\displaystyle\sum_{k=0}^\infty \frac{2^k+3}{5^k}$

5. Determine whether each of the following series converges.
   (a) $\displaystyle\sum_{n=1}^\infty e^{-1/n}$
   (b) $\displaystyle\sum_{n=1}^\infty \frac{1}{(n+3)^{3/2}}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{3}{k(\ln k)^4}$

(d) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{3^k}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^5}{n!}$

6. Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$ converges conditionally, converges absolutely, or diverges.

7. Determine whether the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^{2/3}}$ converges conditionally, converges absolutely, or diverges.

8. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$

9. Find a power series representation for the function
$$f(x) = \frac{x}{1+x^4}.$$
What is the interval of convergence of the power series?

10. Explain what is wrong with the following calculation:
$$\begin{aligned} 1 &= 1+0+0+0+\cdots \\ &= 1+(-1+1)+(-1+1)+(-1+1)+\cdots \\ &= (1+-1)+(1+-1)+(1+-1)+\cdots \\ &= 0+0+0+\cdots \\ &= 0. \end{aligned}$$

11. Suppose that $\{a_n\}$ is a sequence such that $\lim_{n\to\infty} a_n = 3$. Define the sequence $\{b_n\}$ by $b_n = a_n + \frac{2n-3}{5n+1}$.

(a) Does the sequence $\{b_n\}$ converge or diverge? If the sequence converges, find its limit.

(b) Does the series $\displaystyle\sum_{n=1}^{\infty} b_n$ converge or diverge?