Exam 1 Practice

1. Evaluate each of the following integrals.
   (a) $\displaystyle\int \frac{x}{x^2-4}\,dx$
   (b) $\displaystyle\int \frac{1}{x^2-4}\,dx$
   (c) $\displaystyle\int_1^e x\ln x\,dx$
   (d) $\displaystyle\int \frac{\cos(\sqrt{x})}{\sqrt{x}}$

2. Consider the integral
$$I=\int_0^1 \sin(x^2)\,dx.$$
   (a) Estimate the maximum error made in approximating the value of this integral using $n=5$ left rectangular sum approximations.
   (b) How many rectangles would be necessary to guarantee an error of at most $1/200$?

3. Suppose that you wish to estimate
$$I=\int_0^3 e^{x^2}\,dx$$
   using either midpoint or trapezoid sums. Which one would you use if you wanted to be sure to *underestimate* the value of the integral? Justify your answer.

4. Consider the region R in the plane bounded by the graphs of $y=x$ and $y=x^2$.
   (a) Find the area of R.
   (b) Find the volume of the solid obtained by revolving R about the x-axis.
   (c) Find the volume of the solid obtained by revolving R about the line $x=1$.
   (d) Write the length of the boundary of R as a sum of integrals. You do not need to evaluate the integrals.

5. Evaluate $\displaystyle\lim_{x\to 1}\frac{1-x+\ln x}{1+\cos(\pi x)}$.

6. The base of a solid is bounded by $y=x^3$, $x=0$, and $y=1$. Cross-sections of the solid perpendicular to the y-axis are semi-circles. Find the volume of the solid.