Math 112
Homework 9 Solutions

Part 1

11.2 #8: $\frac{\pi^4}{90} - 1 - \frac{1}{2^4}$

11.2 #12: $6 + \frac{1/3}{1-1/3} = \frac{13}{2} = 6.5$

11.2 #14: $\frac{(e/\pi)^3}{1-e/\pi}$

11.2 #16: $\frac{(2/3)^{10}}{1-2/3}$

11.2 #18: $5/2 + 5 = 15/2$

11.2 #24: $S_n = \ln(n+1)$. Since $S_n \to \infty$ as $n \to \infty$, the series diverges.

11.2 #30: $\frac{2^7/5^3}{1-2/5} = 128/75$

11.2 #40: $\frac{1}{1-\ln 2}$

11.2 #42: $\lim_{n\to\infty}(1+\frac{1}{n})^n = e \neq 0$, so the series diverges by the Test for Divergence.

11.2 #52: $\frac{4/7^{10}}{1-1/49} = 1/69177612$

Part 2

11.3 #18: Converges. Use the Comparison Test with the geometric series $\sum_{j=0}^{\infty} \frac{1}{e^j}$.

11.3 #20: Converges. Use the Comparison Test with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^3}$

11.3 #22: Converges. Use the Ratio Test. $\lim_{k\to\infty}\left|\frac{a_{k+1}}{a_k}\right| = \lim_{k\to\infty}\frac{1}{k+1} = 0 < 1$.

11.3 #31: Converges. Use the Comparison Test with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

11.3 #35: Converges. Use the Comparison Test with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^5}$.

11.3 #36: Converges. Use the Integral Test with the improper integral $\int_2^\infty \frac{1}{x(\ln x)^5}$, which converges (use the u-substitution $u = \ln x$).

11.3 #46: Converges. Use the Comparison Test with the p-series $\sum_{m=1}^{\infty} \frac{1}{m^2}$.

11.3 #52: Converges. Use the Ratio Test. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{1}{2(2n+1)} = 0 < 1$.

Part 3

11.4 #8: Converges conditionally. $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k^2-1}$ converges by the Alternating Series Test. $\sum_{k=2}^{\infty} \left|(-1)^k \frac{k}{k^2-1}\right| = \sum_{k=2}^{\infty} \frac{k}{\sqrt{k^2-1}}$ diverges by the Comparison Test with $\sum_{k=2}^{\infty} \frac{1}{k}$.

11.4 #12: Converges absolutely. Use the Ratio Test.

11.4 #20: Converges conditionally. $\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k}}$ converges by the Alternating Series Test. $\sum_{k=1}^{\infty} \left|(-1)^k \frac{1}{\sqrt{k}}\right| = \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ diverges because $p = 1/2 < 1$.

11.4 #24: Converges absolutely. Use the Ratio Test. $\lim_{m\to\infty} \left| \frac{\frac{(-1)^{m+1}(m+1)^3}{2^{m+1}}}{\frac{(-1)^m m^3}{2^m}} \right| = \frac{1}{2} < 1$, so the series converges absolutely by the Ratio Test.