Math 112
Homework 4 Solutions

Section 6.1

6.1 #4: (a). $f(x) = \cos(1/x)$ is increasing on $[1, 3]$. (b) $f(x) = \cos(1/x)$ is concave down on $[1, 3]$.

6.1 #6: (a). $L_{20} \approx 0.158$; $R_{20} \approx 0.122$; $T_{20} \approx 0.14$; $M_{20} \approx 0.139$. (b). L_{20} overestimates I. (c). M_{20} underestimates I.

6.1 #16: Any decreasing function will work. $f(x) = 1/x$ is one choice.

6.1 #18: Any function that is concave down on $[1, 5]$ will work. $f(x) = 1 - x^2$ is one choice.

6.1 #19(a): Upward concavity means that the graph sits beneath the straight line segments that define the trapezoid rule.

6.1 #52: (a). The integrand is increasing until $x = 1$ and decreasing thereafter, so the integrals $\int_0^1 xe^{-x}\,dx$ and $\int_1^4 xe^{-x}\,dx$ can be trapped separately by left and right sums. (b). The integrand is concave down until $x = 2$ and concave up thereafter, so the integrals $\int_0^2 xe^{-x}dx$ and $\int_2^4 xe^{-x}dx$ can be trapped separately by trapezoid and midpoint sums.

6.1 #61: Since f' is negative on $[a, b]$, f is decreasing. Thus, $R_n \leq I \leq L_n$. Since f' is decreasing on $[a, b]$, f is concave down. Thus, $T_n \leq I \leq M_n$. Finally, since T_n and M_n are more accurate than L_n and R_n, we conclude

$$R_n \leq T_n \leq I \leq M_n \leq L_n.$$

Section 6.2

6.2 #6: (a) $K_1 = 24$ works. (b) $K_2 = 6$ works.

6.2 #12: $K_1 = 4$ works, so any $n \geq 1600$ will do.

6.2 #14: $K_1 = 1$ works, so any $n \geq 8100$ will do.

6.2 #16: $K_2 = 11$ works, so any $n \geq 28$ will do.

6.2 #18: $K_2 = 0.45$ works, so any $n \geq 53$ will do.

6.2 #28: (a) For $\int_0^5 e^{-x^2}\,dx$, $K_1 = 0.86$ works, so $n = 1075$. (b) $n = 344$.

Section 7.1

7.1 #14: 1/2

7.1 #18: 1/3

7.1 #24: 343/6

7.1 #44: ≈ 9.07342 (you must evaluate the integral numerically)

7.1 #50: 181/9