Math 112
Homework 1 Solutions

Section 4.2

4.2 #60:

$$\lim_{x\to 0}\frac{\tan(3x)}{\ln(1+x)} = \lim_{x\to 0}\frac{3\sec^2(3x)}{(1+x)^{-1}} = 3$$

4.2 #66:

$$\lim_{x\to 0}\frac{1-x-e^{-x}}{1-\cos x} = \lim_{x\to 0}\frac{e^{-x}-1}{\sin x} = \lim_{x\to 0}\frac{-e^{-x}}{\cos x} = -1$$

4.2 #68:

$$\lim_{x\to\infty} x^2 e^{-x^2/2} = \lim_{x\to\infty}\frac{x^2}{e^{x^2/2}} = \lim_{x\to\infty}\frac{2}{e^{x^2/2}} = 0$$

4.2 #72:

$$\lim_{x\to\pi/2}\frac{\ln(\sin x)}{(x-\pi/2)^2} = \lim_{x\to\pi/2}\frac{\cos x}{2(x-\pi/2)\sin x} = \lim_{x\to\pi/2}\frac{-\sin x}{2\sin x + 2(x-\pi/2)\cos x} = -\frac{1}{2}$$

4.2 #74:

$$\lim_{x\to\infty} x\sin(1/x) = \lim_{x\to\infty}\frac{\sin(1/x)}{1/x} = \lim_{x\to\infty}\cos(1/x) = 1$$

4.2 #83:

$$\lim_{x\to 1}\frac{(f(x))^2-4}{x^2-1} = \lim_{x\to 1}\frac{f(x)f'(x)}{x} = 6$$

4.2 #84:

$$\lim_{x\to 0}\frac{f(x)}{\sin(2x)} = \lim_{x\to 0}\frac{f'(x)}{2\cos(2x)} = \lim_{x\to 0}\frac{f'(0)}{2} = 5.$$

Thus

$$f'(0) = 10.$$

4.2 #88: Let

$$L = \lim_{x\to\infty}(1+x)^{1/x}.$$

Then

$$\begin{aligned}\ln L &= \lim_{x\to\infty} \ln((1+x)^{1/x}) \\ &= \lim_{x\to\infty} \frac{1}{x}\ln(1+x) \\ &= \lim_{x\to\infty} \frac{\ln(1+x)}{x} \\ &= \lim_{x\to\infty} \frac{\frac{1}{1+x}}{1} \\ &= 0.\end{aligned}$$

Thus

$$\ln L = 0,$$

so

$$L = e^0 = 1.$$

Section 5.4

5.4 #32: Let $u = \ln x$. Then

$$\int \frac{\ln x}{x}\, dx = \frac{1}{2}(\ln x)^2 + C$$

5.4 #34: Let $u = x^2$. Then

$$\int xe^{x^2}\, dx = \frac{1}{2}e^{x^2} + C$$

5.4 #40: Let $u = 1 + x^2$. Then

$$\int \frac{x}{\sqrt{1+x^2}}\, dx = \sqrt{1+x^2} + C$$

5.4 #44: Let $u = x^2 + 4x + 5$. Then

$$\int (x+2)(x^2+4x+5)^6\, dx = \frac{1}{14}(x^2+4x+5)^7$$

5.4 #54: Let $u = \sec x$. Then

$$\int \sec x \tan x \sqrt{1+\sec x}\, dx = \frac{2}{3}(1+\sec x)^{3/2} + C$$

5.4 #68:

$$\begin{aligned}\int \frac{e^{\tan x}}{1-\sin^2 x} &= \int \frac{e^{\tan x}}{\cos^2 x}\\ &= \int \sec^2 x e^{\tan x}\end{aligned}$$

Now let $u = \tan x$. Then

$$\int \sec^2 x e^{\tan x} = e^{\tan x} + C$$

5.4 #72: Let $u = \ln x$. Then

$$\begin{aligned}\int_e^{4e} \frac{dx}{x\sqrt{\ln x}} &= 2\sqrt{\ln x}\Big|_e^{4e}\\ &= 2\sqrt{1+2\ln 2} - 2\end{aligned}$$

5.4 #74: Let $u = \cos x$. Then

$$\begin{aligned}\int_{-\pi/2}^{\pi} e^{\cos x}\sin x\, dx &= -e^{\cos x}\Big|_{-\pi/2}^{\pi}\\ &= 1 - e^{-1}\end{aligned}$$

Section 8.1

8.1 #12: Let $u = x$, $dv = \sec x \tan x$. Then $du = dx$ and $v = \sec x$. Then

$$\int x \sec x \tan x\, dx = x\sec x - \ln|\sec x + \tan x| + C.$$

8.1 #26: Note: this problem was not turned in. Let $u = x^n$ and $dv = e^x dx$. Then $du = nx^{n-1}$ and $v = e^x$. Then

$$\int x^n e^x\, dx = x^n e^x - n\int x^{n-1}e^x\, dx.$$

8.1 #32: First make the w-substitution $w = -x^3$. Then $dw = -3x^2 dx$, so $x^2 dx = \frac{-1}{3}dw$.

Thus

$$\begin{aligned}\int x^5 e^{-x^3}\,dx &= \int x^2 x^3 e^{-x^3}\,dx\\ &= \int \frac{1}{3} w e^w\,dw\\ &= \frac{1}{3} w e^w - e^w + C\\ &= -\frac{1}{3}(x^3+1)e^{-x^3} + C\end{aligned}$$

8.1 #53: First make the u-substitution $u = \sin x$. Then $du = \cos x$. Thus

$$\begin{aligned}\int \cos x \ln(\sin x)\,dx &= \int \ln u\,du\\ &= u\ln u - u\\ &= (\sin x)\ln(\sin x) - \sin x + C.\end{aligned}$$

8.1 #54: First make the u-substitution $u = -x^2$. Then

$$\int x^5 e^{-x^2}\,dx = \int x x^4 e^{-x^2}\,dx = -\frac{1}{2}\int u^2 e^u\,du.$$

Then use integration by parts twice to obtain

$$\int x^5 e^{-x^2}\,dx = -(x^4/2 + x^2 + 1)e^{-x^2} + C.$$

8.1 #60: Note: this problem was not turned in.

(a) $I_0 = \int_0^1 e^x\,dx = e^x\Big|_0^1 = e - 1$

(b) Using the reduction formula, $I_1 = \int_0^1 xe^x\,dx = xe^x\Big|_0^1 - I_0 = 1$

(c) Using the reduction formula, $I_2 = x^2e^x\Big|_0^1 - 2I_1 = e - 2$

(d) The reduction formula implies that $I_n = e - nI_{n-1}$. Thus $I_3 = e - 3I_2 = 6 - 2e$, $I_4 = 9e - 24$, and $I_5 = 120 - 44e$.