Math 112
Homework 11 Solutions

11.6 #7:

$$\begin{aligned} f(x) &= (1+x)^{-2} \\ &= \frac{d}{dx}(-(1+x)^{-1}) \\ &= -\frac{d}{dx}\left(\sum_{k=0}^{\infty}(-x)^k\right) \\ &= \sum_{k=1}^{\infty} k(-x)^{k-1} \\ &= 1 - x + 3x^2 - 4x^3 + \cdots \end{aligned}$$

11.6 #20:

$$\begin{aligned} \ln(1+x^2) &= \int \frac{2x}{1+x^2}\,dx \\ &= \int 2x \cdot \sum_{k=0}^{\infty}(-x^2)^k\,dx \\ &= \int \sum_{k=0}^{\infty}(-1)^k 2x^{2k+1}\,dx \\ &= \sum_{k=0}^{\infty}\int (-1)^k 2x^{2k+1}\,dx \\ &= \sum_{k=0}^{\infty}(-1)^k \frac{2x^{2k+1}}{2k+2} + C \\ &= \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{k+1} + C \end{aligned}$$

Setting $x = 0$, we obtain $C = \ln(1) = 0$. Thus,

$$\ln(1+x^2) = \sum_{k=0}^{\infty}(-1)^k \frac{x^{2k+1}}{k+1}.$$

11.6 #22:

$$\begin{aligned}
\ln\left(\frac{1+x}{1-x}\right) &= \ln(1+x) - \ln(1-x) \\
&= \int \frac{1}{1+x}\,dx + \int \frac{1}{1-x}\,dx \\
&= \int \sum_{k=0}^{\infty} (-1)^k x^k\,dx + \int \sum_{k=0}^{\infty} x^k\,dx \\
&= \sum_{k=0}^{\infty} \int (-1)^k x^k\,dx + \sum_{k=0}^{\infty} \int x^k\,dx \\
&= \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1} + C_1 + \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} + C_2
\end{aligned}$$

Setting $x = 0$, we obtain $C_1 + C_2 = \ln(1) = 0$. Thus,

$$\begin{aligned}
\ln\left(\frac{1+x}{1-x}\right) &= \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1} + \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} \\
&= 2\sum_{k=0}^{\infty} \frac{x^{2k+1}}{2k+1}
\end{aligned}$$