

# An Exploration of $n$ -Trigonometric Functions: Generalizations of Sine and Cosine

Alex Beckwith

Every first-year calculus student learns that the derivative of  $\sin(x)$  is  $\cos(x)$  and the derivative of  $\cos(x)$  is  $-\sin(x)$ . And from this a student might notice that the second derivative of  $\sin(x)$  is  $-\sin(x)$  and similarly so for cosine. In fact,  $\sin(x)$  and  $\cos(x)$  are solutions of the differential equation  $f^{(2)}(x) = -f(x)$ . In this project, we begin by considering the differential equation  $f^{(n)}(x) = -f(x)$  for a fixed  $n$ . We use the so-called power series method to identify  $n$  solutions to this differential equation. These functions are called the  $n$ -trigonometric functions (thus,  $\sin(x)$  and  $\cos(x)$  are 2-trigonometric functions). In this project we study geometric and analytic properties of the  $n$ -trigonometric functions.

Monday October 22

3:10PM

RBH 311