Math 112 Fall 2009
Taylor Polynomials: Motivation

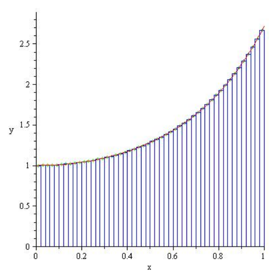
An important use of Taylor polynomials is that they approximate a given (non-polynomial) function f near a given domain point x_0. If f happens to be complicated, inconvenient, or poorly understood, then having a close polynomial approximation p can reduce clutter and simplify calculations.
Let us recall the definition of the Taylor polynomial.

Let f be any function whose first n derivatives exist at $x = x_0$. The Taylor polynomial of order n, based at $x = x_0$, is defined by

$$P_n(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \frac{f'''(x_0)}{3!}(x - x_0)^3 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n.$$

Here is how the Taylor polynomials can come handy.
Example: Find $\int_0^1 e^{x^2} dx$.
Well.. We have seen this example many times so far, and we all know that this is not an easy integral.
We know that none of the techniques we learned applies to solve this integral symbolically. What about a Riemann sum approximation?

1. Let us approximate this integral using midpoint sum with 50 subintervals.



   The next task is to make use of Taylor polynomials. We have already seen how to approximate the function $f(x) = e^x$ using Taylor polynomials.

2. Write a degree 4 Taylor polynomial approximating $f(x) = e^x$ around $x = 0$.

3. Can you get the polynomial approximation for e^{x^2} using part 2?

4. Integrate the above result in part 3 and compare your answer with part 1.