Tests for Convergence of Series

First we will go over some important facts, that are necessary for you to know if you want to become an expert on series.

1. What is a geometric series, when is a geometric series convergent? If a geometric series is convergent to which value does it converge to?

2. What is a p-series, when does a p-series converge?

3. Define the harmonic series. Is it convergent or divergent?

4. What is a telescoping sum? How do you find the limit of such a sum?

5. Rank the following functions from fastest to slowest in terms of how fast they go to infinity as n gets large.

$$n^n, e^n, 3^n, n!, x^2, x^4 + 1, \ln(x)$$

Review of the convergence tests for series

1. **The nth term test for divergence, or divergence detector**

   Disadvantage: Divergence detector does not say that if the sequence of terms goes to zero, then the series converges. This is a test for divergence only. It never tells you anything about convergence. One thing you can conclude from this test is that for a series to be convergent it is necessary that the sequence of terms go to zero but this is not sufficient. How fast they go to zero is extremely important. For example consider $\Sigma\frac{1}{k}$ and $\Sigma\frac{1}{k^2}$.

2. **Comparison Test**

   Advantages: Conclusive test. If you can find the right series to use for comparison, this test always give an answer.
   Disadvantages: If the series does not easily compare to one of our "known" series, then you are out of luck. You also kind of have an intuition about whether or not the series you are testing is convergent or not so you can find the right series for comparison.
   Rule of thumb: This test works best where there are just powers of n in the numerator and the denominator. Easy comparison can be made to the p-series.

3. **Limit Comparison Test**

   Advantages: This test is easier to apply than the comparison test.
   Disadvantages: Same as comparison test disadvantages and it can also yield inconclusive results, when the limit doesn't exist or when the cases in the statement of the test are not satisfied.
   Rule of thumb: This test works best where there are just powers of n in the numerator and the denominator. Easy comparison can be made to the p-series.

4. **Integral test**

   Advantages: You don't have to know ahead of the time whether the sequence is converges or diverges and the test is conclusive.
   Disadvantages: You have to be able to associate your series with an appropriate function that satisfies the

hypothesis of the test.
Rule of thumb: Use the integral test only if you look at the function and immediately know how to compute the integral. Otherwise do not waste your time, another test is likely to work.

5. **Ratio test, Generalized ratio test**

Advantages: You don't have to know ahead of the time whether the sequence is converges or diverges. You don't have to find associated functions or comparison series. When the test works it is one of the easiest to apply.
Disadvantages: It may be inconclusive.
Rule of thumb: This test works well for series whose terms have factorials or exponentials. It never works on rational functions (ratio of two polynomials) , do not even bother to try it.

6. **Alternating series test**

7. **Absolutely and conditionally convergence**

Advantages: You can easily detect the series for which the last two tests are handy.