

Math 112 Fall 2009
Improper integrals

What is an improper integral?

Examples: $\int_1^\infty \frac{dx}{x^2}$, $\int_1^\infty \frac{dx}{x}$, $\int_\infty^\infty \frac{dx}{1+x^2}$, $\int_0^\infty e^{-x} dx$, $\int_0^1 \frac{dx}{x^2}$, $\int_0^1 \frac{dx}{\sqrt{x}}$, $\int_0^\infty \frac{dx}{\sqrt{x+x^2}}$.

What are the “impropriety” in those integrals?

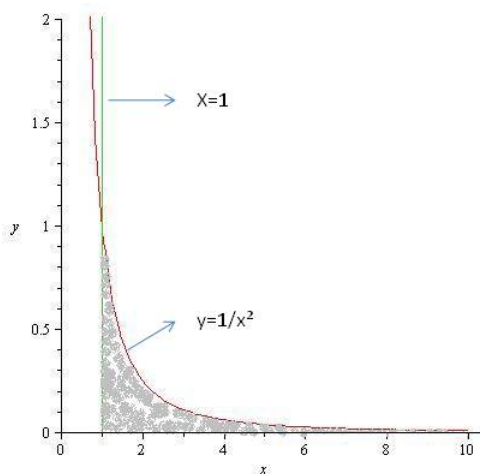
1. Infinite intervals
2. Infinite integrands

Determine the impropriety for all the examples above.

Convergence and divergence: Informally improper integrals that have a sensible numerical value are called convergent and they are called divergent otherwise.

We start with a few basic examples:

Example 1: Make sense of $\int_1^\infty \frac{dx}{x^2}$ and determine whether this integral is convergent or not.



Example 2: Does $\int_1^\infty \frac{dx}{x}$ converge?

Example 3: Does $\int_{-\infty}^\infty \frac{dx}{1+x^2}$ converge? If so, to what value?

Example 4: Does $\int_0^1 \frac{dx}{x^2}$ converge? If so, to what value?

Example 5: Does $\int_{-1}^1 \frac{dx}{x^2}$ converge? If so, to what value?

Example 6: Does $\int_0^\infty \cos x dx$ converge or diverge?

Example 7: Determine for which values of the exponent, p , the improper integral $\int_1^{\infty} \frac{dx}{x^p}$ converges/diverges.

Example 8: Determine for which values of the exponent, p , the improper integral $\int_0^1 \frac{dx}{x^p}$ converges/diverges.

Example 9: Determine for which values a , the improper integral $\int_0^{\infty} e^{-ax} dx$ converges/diverges.

Example 10: Does $\int_1^{\infty} \frac{dx}{x^5+1}$ converge or diverge?

This example will lead to the next section: Detecting Convergence, Comparison test for integrals