Group Homework
Math 112, Fall 2009
Selin Kalaycioglu

The goal of this project is to prove the second part of the theorem about error bounds for approximating sums:

Theorem: Let $I = \int_a^b f(x)dx$, and let M_n and T_n denote midpoint and trapezoid sums for I, each with n equal subdivisions. Let K_2 be a constant such that $|f''(x)| \leq K_2$ for all x in $[a, b]$. Then

$$|I - T_n| \leq \frac{K_2(b-a)^3}{12n^2} \text{ and } |I - M_n| \leq \frac{K_2(b-a)^3}{24n^2}.$$

In the following I tried to outline the steps that will give you enough hints to prove the theorem.

1. Find a function $q(x)$ for which $|q''(x)| \leq K_2$ for all $x \in \mathbb{R}$.
2. Suppose $f(0) = 0$ and $f'(0) = 0$. Draw a sketch of the graph of $f(x)$ and $q(x)$ on the same coordinate system. Label the interval $[a, b]$ on your graph noting the assumption that $0 < a < b$.
3. The first step is to estimate how much error M_n and T_n can commit, at worst, over one subinterval. To simplify the algebra, we take $[0, h]$ as our subinterval. First compute the exact integral of $q(x)$ over $[0, h]$ and denote that by I_h.
4. Now compute the midpoint and trapezoid estimates over the same interval for $q(x)$ and call them M_h and T_h respectively.
5. Compute $I_h - M_h$, and $I_h - T_h$. Do these expressions convinve you that you are on the right track?
6. Now that you have found the worst-case error committed by each method over one subinterval, estimate the worst case error, over all n subintervals on $[a, b]$ for both the midpoint and the trapezoid approximations. Write a few sentences to conclude why this proves the theorem.