More Integration techniques

You can get in groups and give me one written work if you want. I am not assigning groups since it might be hard to get together over the reading days. However if you know that someone is in town you would like to work with, I strongly encourage you to work together. It will be more fun that way :)

1. Use integration by parts on $\int_0^x f''(t)(x-t)dt$ to show that

$$f(x) - f(0) = f'(0)x + \int_0^x f''(t)(x-t)dt.$$

2. Find $\int \frac{1}{x^2+6x+14}dx$.

3. Find $\int e^t \sin(5t+7)dt$.

4. (Adapted from Calculus,Hughes-Hallett, et al.) A rumor is spread in Kenyon College campus. For $0 < a < 1$ and $b > 0$, the time t at which a fraction p of the school population has heard the rumor is given by

$$t(p) = \int_a^p \frac{b}{x(1-x)}dx.$$

   (a) Evaluate the integral to find an explicit formula for $t(p)$. Write your answer so it has only one ln term.

   (b) At time $t = 0$ one percent of the Kenyon population ($p = 0.01$) has heard the rumor. What is a?

   (c) At time $t = 1$ half the school population has heard the rumor. What is b?

(d) At what time has 90% of the school population heard the rumor?

5. The moment generating function, $m(t)$, which gives useful information about the normal distribution of statistics is defined by

$$m(t) = \int_{-\infty}^{\infty} e^{tx} \frac{e^{-x^2/2}}{\sqrt{2\pi}}dx.$$

   Find a formula for $m(t)$.

   Hint: Combine the exponential terms. Complete the square of the exponent of e and then use the fact that $\int_{-\infty}^{\infty} e^{-x^2/2}dx = \sqrt{2\pi}$.