Abstract II Spring 2010
Algebraic Extensions-Section 31 Part 2

Review: Degrees of extensions, minimal polynomial of an algebraic element...
Example: $\mathbb{Q}(2^{1/2}, 2^{1,3})$

Theorem 0.1. *Let E be an algebraic extension of a field F. Then there exists a finite number of elements $\alpha_1, \alpha_2, \ldots, \alpha_n$ in E such that $E = F(\alpha_1, \alpha_2, \ldots, \alpha_n)$ if and only if E is a finite dimensional vector space over F.*

Proof:

Algebraically closed fields and Algebraic Closures

Fact: If α, β are algebraic over F, then so are $\alpha \pm \beta$, $\alpha\beta$ and α/β if $\beta \neq 0$.

Theorem 0.2. *Let E be an extension field of F. Then*

$$\bar{F}_E = \{\alpha \in E | \alpha \text{ algebraic over } F\}$$

is a subfield of E, the algebraic closure of F in E.

Proof:

Definition 0.3. *A field F is algebraically closed if every nonconstant polynomial in $F[x]$ has a zero in F.*

Theorem 0.4. *A field F is algebraically closed if and only if every nonconstant polynomial in $F[x]$ factors in $F[x]$ into linear factors.*

Proof:

Theorem 0.5. *Every field F has an algebraic closure, that is an algebraic extension $\bar{F}$ that is algebraically closed.*

Theorem 0.6. *(Fundamental Theorem of Algebra) The field $\mathbb{C}$ of complex numbers ias an algebraically closed field.*