Abstract II Spring 2010
Algebraic Extensions

Goal: Study extensions of F containing only elements algebraic over F.

Definition 0.1. *An extension field E of a field F is an algebraic extension of F if every element in E is algebraic over F.*

Definition 0.2. *If an extension field E of a field F is of finite dimensional n as vector space over F, then E is a finite extension of degrenn n over F. We denote the degree n of E over F by $[E : F]$.*

Note that this definition does not require E to be a finite field, it just states that as a vector space over F, E has finite dimension.

Recall that a finite extension field E of a field F is an algebraic extension of F.

Theorem 0.3. *If E is a finite extension field of a field F, and K is a finite extension field of E, then K is a finite extension of F, and*

$$[K : F] = [K : E][E : F]$$

Proof:

Corollary 0.4. *If E is an extension field of F, $\alpha \in E$ is algebraic over F, and $\beta \in F(\alpha)$, then $deg(\beta, F)$ divides $deg(\alpha, F)$.*

Proof:

Examples: