Math 112 Quiz 7 – **Last One!**
5/1/2009 – **Last Day!**

Name Solutions

Directions: Answer the questions below using the techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

1. Does this series diverge, converge absolutely, or converge conditionally? Justify your response.

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$$

Note that:

$\lim_{n\to\infty} \frac{(-3)^n}{n^3}$ does not exist, since $(-3)^n$ oscillates.

Thus, this series fails the n^{th}-term test and does not converge.

2. Find the radius and the interval of convergence of the series

$$\sum_{n=1}^{\infty} n!(2x-1)^n$$

Ratio Test:

$$\lim_{n\to\infty} \frac{(n+1)!(2x-1)^{n+1}}{n!(2x-1)^n} = \lim_{n\to\infty} (n+1)(2x-1) = \infty$$

So this series converges only at the point $x = \frac{1}{2}$; its radius of convergence is 0.

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3. Find the radius and the interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{(x+5)^n}{n\ln(n)}$$

Ratio Test:

$$\lim_{n\to\infty} \frac{\frac{|x+5|^{n+1}}{(n+1)\ln(n+1)}}{\frac{|x+5|^n}{n\ln n}} = \lim |x+5| \cdot \underbrace{\frac{n\ln n}{(n+1)\ln(n+1)}}_{\to 1} = |x+5$$

This series converges if $|x+5| < 1$, so radius of convergence is 1.

$$|x+5| < 1 \iff -6 < x < -4$$

Check endpoints:

$x=-6$: $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ converges by AST.

$x=-4$: $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges, by integral test:

$$\int_2^{\infty} \frac{1}{x\ln x}\,dx \qquad \text{Let } u = \ln x,\ du = \frac{1}{x}du$$

$$= \int \frac{1}{u}\,du = \ln|u|\Big] = \ln(\ln|x|)\Big]_2^{\infty}$$

$$= \lim_{t\to\infty} \ln(\ln(x))\Big]_2^t = \lim_{t\to\infty}\left[\ln(\ln(t)) - \ln(\ln(2))\right]$$

$$= \infty$$

So this series has an interval of convergence of

$$-6 \le x < -4$$