Math 112 Quiz 6
4/13/2009

Name _____

Directions: Answer the questions below using the techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

S:11

1. Find the limit of the sequence, or explain why it does not exist.

   a. $a_m = m^2 e^{-m}$ (11.1 #13)

$a_m = \frac{m^2}{e^m}$ $\lim a_m = \lim \frac{m^2}{e^m} = \frac{\infty}{\infty}$

$= \lim \frac{2m}{e^m} = \lim_{m\to\infty} \frac{2}{e^m} = \frac{2}{\infty} = 0.$

   b. $a_k = \frac{k!}{(k+1)!}$ (#15)

$\lim a_k = \lim \frac{k!}{(k+1)!} = \lim_{k\to\infty} \frac{1}{k+1} = 0.$

   c. $a_n = \frac{\cos^2 n}{2^n}$ (#12 from wkst)

$\lim a_n$: $\quad 0 \le \cos^2 n \le 1$

$\Rightarrow \frac{0}{2^n} \le \frac{\cos^2 n}{2^n} \le \frac{1}{2^n}$

$\frac{0}{2^n} \to 0$, $\frac{1}{2^n} \to 0$

$\to 0.$

2. Determine whether the series below converges or diverges. If it converges, find its limit. (11.3 #49)

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$$\frac{3}{10} - \frac{3}{20} + \frac{3}{40} - \frac{3}{80} + \frac{3}{160} - \frac{3}{320} + \cdots$$

$\sum 3\left[\frac{1}{10} - \frac{1}{20} + \frac{1}{40} - \cdots\right] = 3/10\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32}\right]$

$\sum_{k=0}^{\infty} \frac{3}{10}\left(-\frac{1}{2}\right)^k = \frac{3/10}{1 + 1/2}$ $\qquad \sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k$

($\frac{3}{10}$ = a, $-\frac{1}{2}$ = r)

$= \frac{3}{10} \cdot \frac{2}{3} = \frac{1}{5}.$

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3. Suppose that the partial sums of the series $\sum_{k=1}^{\infty} a_k$ are given by

$$S_n = \sum_{k=1}^{n} a_k = 5 - \frac{3}{n} \quad (11.2\ \#25)$$

a. Evaluate S_{100}.

+2 $S_{100} = 5 - \frac{3}{100} = \frac{497}{100} = 4.97$

b. Evaluate $\sum_{k=1}^{\infty} a_k$.

+3 $\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(5 - \frac{3}{n}\right) = 5.$

So $\sum a_k$ converges to 5.

c. Eval. $\lim_{k\to\infty} a_k$.

+3 Since $\sum a_k$ converges, $\lim_{k\to\infty} a_k = 0$.

5:14.

cancel all from ax.