Math 112 Quiz 5
4/3/2009

Name Solutions

Directions: Answer the questions below using the integration techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

1. Use an antiderivative to evaluate the improper integral

$$\int_3^\infty \frac{x}{(x^2-4)^3}\,dx = \lim_{t\to\infty} \int_3^t x(x^2-4)^{-3}\,dx \quad +1$$

Let $u = x^2-4$, $du = 2x\,dx$. +2

$$= \frac{1}{2}\lim_{t\to\infty}\int_{x=3}^{x=t} u^{-3}\,du = \frac{1}{2}\lim\left(-\frac{1}{2}u^{-2}\right]_{x=3}^{x=t} = \frac{1}{2}\lim_{t\to\infty}\left(-\frac{1}{2}(x^2-4)^{-2}\right]_3^t \quad +1$$

$$= \lim_{t\to\infty}\left(-\underbrace{\frac{1}{4(t^2-4)^2}}_{\downarrow\ 0} + \frac{1}{4(9-4)^2}\right) = \frac{1}{4(25)} = \frac{1}{100} \quad +1$$

+1.5

2. Let S be the region between the graphs $y = 1/x$ and $y = 1/\sqrt{x}$ between $x=0$ and $x=1$. Does S have finite area? Justify your answer.

$S = \int_0^1 \left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right)dx$, which is discontinuous at $x=0$.

+1.5

$$= \lim_{t\to 0^+}\int_t^1 x^{-1} - x^{-1/2}\,dx = \lim\left(\ln(x) - 2x^{1/2}\right]_t^1$$

+1 +1

$$= \lim_{t\to 0^+}\left(\ln(1) - 2 - \underset{\downarrow\ -\infty}{\ln(t)} + \underset{\downarrow\ 0}{2t^{1/2}}\right) \longrightarrow \text{diverges} \quad +1$$

+1.5

So S does not have finite area. +1

3. Use the Comparison Theorem to determine whether the integral below is convergent or divergent.

$$I = \int_1^\infty \frac{x}{\sqrt{1+x^6}}\,dx$$

Note that $1+x^6 > x^6$, which means $\sqrt{1+x^6} > \sqrt{x^6}$

$$\Rightarrow \frac{1}{\sqrt{1+x^6}} < \frac{1}{\sqrt{x^6}} = \frac{1}{x^3}$$

+2.5

So $I < \int_1^\infty \frac{x}{x^3}\,dx = \int_1^\infty \frac{1}{x^2}\,dx$

$$= \lim_{t\to\infty}\int_1^t x^{-2}\,dx = \lim\left(-x^{-1}\right]_1^t = \lim_{t\to\infty}\left(-\underset{\downarrow\ 0}{\frac{1}{t}} + \frac{1}{1}\right)$$

$= 1$ +3

So the integral I converges to something less than 1. +1