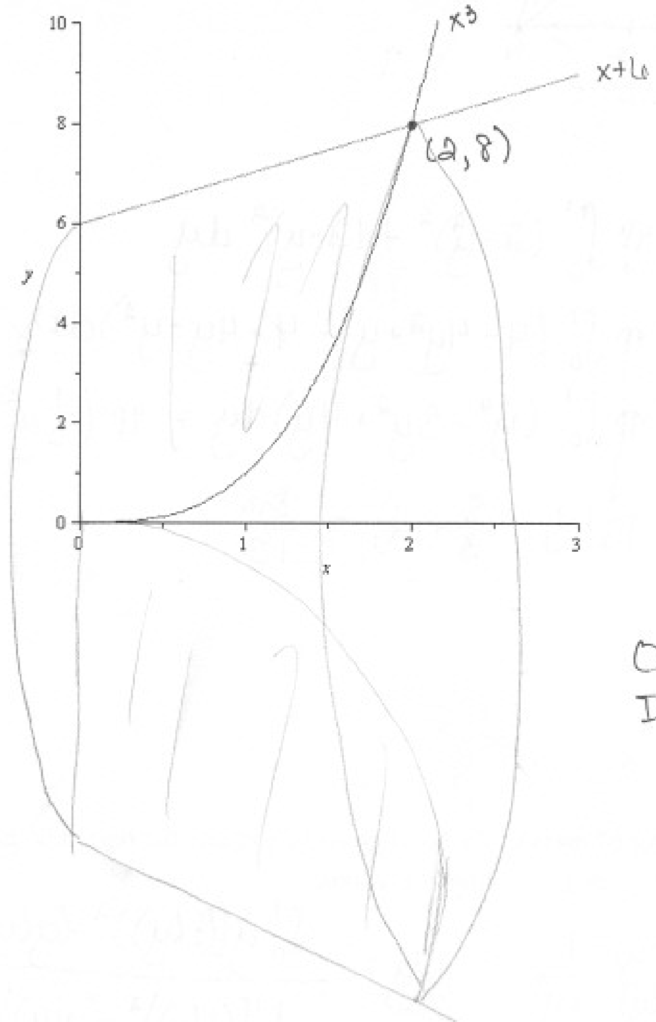
Math 112 Quiz 4
3/18/2009

Name Solutions

Directions: Answer the questions below using the integration techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

1. Below are the graphs of $y = x + 6$ and $y = x^3$. Find the volume of the solid that is formed when the area between these two curves is revolved around the x-axis.



Outer radius: $x+6$
Inner radius: x^3

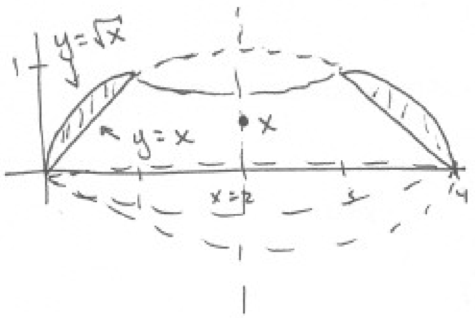
$$V = \pi \int_0^2 (x+6)^2 - (x^3)^2\,dx = \pi \int_0^2 (x^2 + 12x + 36 - x^6)\,dx$$

$$= \pi \left(\tfrac{1}{3}x^3 + 6x^2 + 36x - \tfrac{1}{7}x^7 \right]_0^2 = \pi \left(\frac{8}{3} + 6 \cdot 4 + 72 + \frac{-2^7}{7} \right)$$

$$= \frac{1688\pi}{21} \approx 252.52$$

Math 112 Quiz 4
3/18/2009 Name ______________________________

2. Find the volume of the solid obtained by revolving the region bound by $y = x$ and $y = \sqrt{x}$ about the line $x = 2$.



Outer radius: $y = \sqrt{2-x} \Rightarrow x = 2 - y^2$

Inner radius: $y = 2 - x \Rightarrow x = 2 - y$

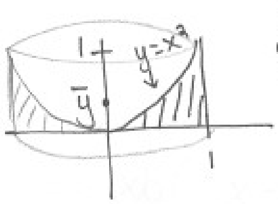
$$V = \pi \int_0^1 (2-y^2)^2 - (2-y)^2 \, dy$$

$$= \pi \int_0^1 (\cancel{4} - 4y^2 + y^4 - \cancel{4} + 4y - y^2)\,dy$$

$$= \pi \int_0^1 (y^4 - 5y^2 + 4y)\,dy = \pi \left(\tfrac{1}{5}y^5 - \tfrac{5}{3}y^3 + 2y^2\right]_0^1$$

$$= \pi\left(\tfrac{1}{5} - \tfrac{5}{3} + 2\right) = \frac{8\pi}{15}$$

3. Find the center of mass of the solid given by rotating the region bound by the curves $y = x^2$, $y = 0$, and $x = 1$ about the y-axis.



$f(y) = 1$

$g(y) = \sqrt{y}$

$$\bar{y} = \frac{\int_0^1 y[(f(y))^2 - (g(y))^2]\,dy}{\int_0^1 [(f(y))^2 - (g(y))^2]\,dy}$$

$$= \frac{\int_0^1 y(1^2 - (\sqrt{y})^2)\,dy}{\int_0^1 (1^2 - (\sqrt{y})^2)\,dy} = \frac{\int_0^1 y(1-y)\,dy}{\int_0^1 (1-y)\,dy}$$

$$= \frac{\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3\Big]_0^1}{y - \tfrac{1}{2}y^2\Big]_1^1} = \frac{\tfrac{1}{2} - \tfrac{1}{3}}{1 - \tfrac{1}{2}} = \frac{\tfrac{1}{6}}{\tfrac{1}{2}} = \frac{1}{3}$$