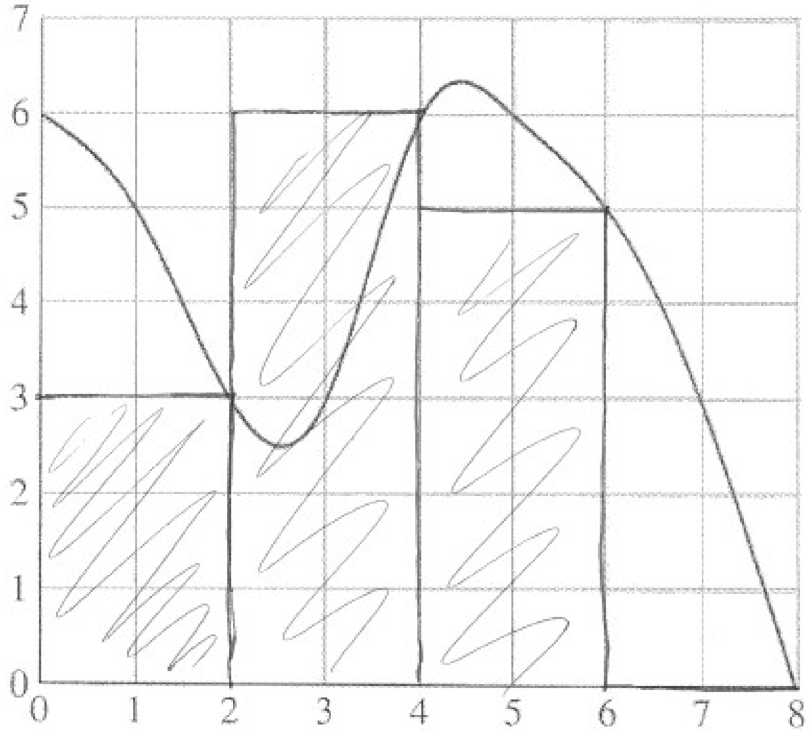
Math 112 Quiz 3
2/11/2009

Name Solutions

Directions: Answer the questions below using the integration techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

1. The graph of $f(x)$ is below. Estimate the value of $\int_0^8 f(x)dx$ using a right sum with four equal subintervals. Calculate the estimate *and* sketch the approximating rectangles on the graph below.



$$R_4 = 2\cdot f(2) + 2\cdot f(4) + 2\cdot f(6) + 2\cdot f(8)$$
$$= 2\cdot 3 + 2\cdot 6 + 2\cdot 5 + 2\cdot 0$$
$$= 28$$

2. Evaluate $\lim_{n\to\infty} \frac{1+2+3+\cdots+n}{n^2}$

$$1+2+3+\cdots+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$$

So $$\lim_{n\to\infty} \frac{1+2+\cdots+n}{n^2} = \lim_{n\to\infty} \frac{n^2+n}{2n^2} = \lim_{n\to\infty} \frac{1+1/n}{2} = \frac{1}{2}$$

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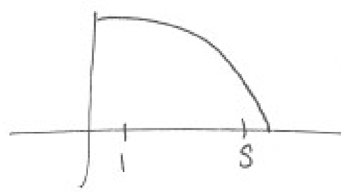
3. Let $I = \int_1^5 f(x)dx$. Give an example (graphical or symbolic) of a function $f(x)$ such that the inequality $T_n < I$ is true for all $n \geq 1$. Make sure to identify the property (or properties) of $f(x)$ that guarantee that the inequality is true.

When $f(x)$ is concave down on $[1,5]$, T_n underestimates I.

An example of a concave down function is:



4. Consider the integral $I = \int_0^1 (1+x^2)^{-1} dx$. Using the error bound formula

$$|I - L_n| \leq \frac{K_1(b-a)^2}{2n},$$

find a value of n that guarantees L_n approximates I to within ±0.005. (Note: $0.005 = \frac{1}{200}$)

$$f(x) = \frac{1}{1+x^2} \Rightarrow |f'(x)| = \left|\frac{-2x}{(1+x^2)^2}\right| = |-2x| \cdot \left|\frac{1}{(1+x^2)^2}\right|$$

$$\leq |-2 \cdot 1| \cdot \left|\frac{1}{(1+0^2)^2}\right| = 2 \cdot 1 = 2 = K_1$$

So $|I - L_n| \leq \frac{2(1-0)^2}{2n} = \frac{2}{2n} = \frac{1}{n} \leq 0.005$

$n \geq 200$