Math 112 Quiz 2
1/28/2009

Name Solutions

Directions: Answer the questions below using the integration techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!
The following formula may be helpful:

$$\sin^2(x)=\frac{1}{2}(1-\cos(2x)) \qquad \cos^2(x)=\frac{1}{2}(1+\cos(2x))$$

1. Find the antiderivative: $\int \sin^2(3x)dx$

Let $u=3x$, $du=3dx$

$$\int \sin^2(3x)dx=\frac{1}{3}\int \sin^2 u\,du=\frac{1}{3}\cdot\frac{1}{2}\int(1-\cos(2u))du$$
$$=\frac{1}{6}\int 1du-\frac{1}{6}\int\cos(2u)du=\frac{1}{6}u-\frac{1}{6}\cdot\frac{1}{2}\sin(2u)+C$$
$$=\frac{1}{6}\cdot 3x-\frac{1}{12}\sin(2\cdot 3x)=\frac{x}{2}-\frac{\sin(6x)}{12}+C$$

2. Find the antiderivative: $\int \frac{x}{3x+2}dx$

To make this proper:

$$3x+2\overline{\big)\,x+0}\quad \frac{1}{3}$$
$$\underline{x+2/3}$$
$$-2/3$$

$\Rightarrow$ So $\frac{x}{3x+2}=\frac{1}{3}-\frac{2/3}{3x+2}$

$$\int\frac{x}{3x+2}dx=\int\left(\frac{1}{3}-\frac{2}{3}\cdot\frac{1}{3x+2}\right)dx$$
$$=\frac{1}{3}x-\frac{2}{3}\int\frac{1}{3x+2}dx$$

Let $u=3x+2$
$du=3dx$

$$=\frac{1}{3}x-\frac{2}{3}\cdot\frac{1}{3}\int\frac{1}{u}du$$
$$=\frac{1}{3}x-\frac{2}{9}\ln|u|+C=\frac{1}{3}x-\frac{2}{9}\ln|3x+2|+C$$

3. Find the antiderivative: $\int\frac{5x^2+3x-2}{x^3+2x^2}dx$

$$\frac{5x^2+3x-2}{x^2(x+2)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$$
$$5x^2+3x-2=Ax(x+2)+B(x+2)+Cx^2=Ax^2+2Ax+Bx+2B+Cx^2$$

$5=A+C$ $\quad 5=2+C \Rightarrow C=3$
$3=2A+B$ $\quad 2A-1=3\Rightarrow A=2$
$-2=2B$ $\quad\Rightarrow B=-1$

$$\text{So }\int\frac{5x^2+3x-2}{x^2(x+2)}dx=\int\frac{2}{x}dx-\int\frac{1}{x^2}dx+\int\frac{3}{x+2}dx$$
$$=2\ln|x|+\frac{1}{x}+3\ln|x+2|+C$$

2. Alternative method: IBP

$$\int \frac{x}{3x+2}\,dx$$

$u = x \quad \Rightarrow \quad du = dx$

$dv = \frac{1}{3x+2}\,dx \quad \Rightarrow \quad v = \frac{1}{3}\ln|3x+2|$

$$\int \frac{x}{3x+2}\,dx = \frac{1}{3}x\ln|3x+2| - \frac{1}{3}\int \ln|3x+2|\,dx$$

Let $u = 3x+2$, $du = 3dx$

$$= \frac{1}{3}x\ln|3x+2| - \frac{1}{3^2}\int \ln u\,du$$

$$= \frac{1}{3}x\ln|3x+2| - \frac{1}{9}(u\ln u - u) + C$$

$$= \frac{1}{3}x\ln|3x+2| - \frac{1}{9}(3x+2)\ln|3x+2| + \frac{1}{9}(3x+2) + C$$