Math 112 Quiz 1
1/21/2009

Name Solutions

Directions: Answer the questions below using the integration techniques we have discussed so far. No aids (calculators, Maple, etc.) are allowed on this quiz. To receive full credit, you must **show your work**. Simplify answers as much as possible.
Good luck!

1. Evaluate the integral: $\int_1^4 (5x + x^{3/2})\,dx$

$$\int_1^4 (5x+x^{3/2})\,dx = \int_1^4 5x\,dx + \int_1^4 x^{3/2}\,dx = \frac{5}{2}x^2\Big]_1^4 + \frac{2}{5}x^{5/2}\Big]_1^4$$
$$= \frac{5}{2}(16-1) + \frac{2}{5}(32-1) = \frac{499}{10} = 49.9$$

2. Find the antiderivative of $\int x\sqrt{3-2x}\,dx$

$$\int x\sqrt{3-2x}\,dx = \quad \text{Let } u = 3-2x \Rightarrow du = -2\,dx, \quad x = \frac{3-u}{2}$$
$$= -\frac{1}{2}\int \frac{3-u}{2}\,u^{1/2}\,du = -\frac{1}{4}\int (3-u)u^{1/2}\,du = -\frac{1}{4}\int (3u^{1/2} - u^{3/2})\,du$$
$$= -\frac{1}{4}\left[\frac{2}{3}\cdot 3u^{3/2} - \frac{2}{5}u^{5/2}\right] + C = -\frac{1}{4}\left[2(3-2x)^{3/2} - \frac{2}{5}(3-2x)^{5/2}\right] + C$$
$$= \frac{1}{10}(3-2x)^{5/2} - \frac{1}{2}(3-2x)^{3/2} + C$$

3. Find the antiderivative of ~~$\int (\ln x)^2\,dx$~~ $\int x\sin(3x)\,dx$. (Hint: Use integration by parts)

$u = x \Rightarrow du = dx$
$dv = \sin(3x)\,dx \Rightarrow v = -\frac{1}{3}\cos(3x)$

$$\int x\sin(3x)\,dx = -\frac{1}{3}x\cos(3x) + \frac{1}{3}\int \cos(3x)\,dx$$
$$= -\frac{1}{3}x\cos(3x) + \frac{1}{9}\sin(3x) + C$$

4. Evaluate the integral: $\int_1^e \frac{\sin(\ln x)}{x}\,dx$

$$\int_1^e \frac{\sin(\ln x)}{x}\,dx \quad \text{Let } u = \ln x \Rightarrow du = \frac{1}{x}\,dx$$
$$= \int_{x=1}^{x=e} \sin(u)\,du = -\cos(u)\Big]_{x=1}^{x=e} = -\cos(\ln x)\Big]_1^e$$
$$= -\cos(\ln e) + \cos(\ln 1) = \cos(0) - \cos(1) = 1 - \cos(1)$$

2. Alternative method : IBP

Let $u = x$ $\quad dv = (3-2x)^{1/2}dx$ $\Rightarrow$ $du = dx$ $\quad v = \frac{2}{3} \cdot -\frac{1}{2}(3-2x)^{3/2}$

$$\int x\sqrt{3-2x}\,dx = -\frac{x}{3}(3-2x)^{3/2} + \frac{1}{3}\int (3-2x)^{3/2}dx$$

$$= -\frac{x}{3}(3-2x)^{3/2} + \frac{1}{3} \cdot \frac{2}{5}(3-2x)^{5/2} \cdot -\frac{1}{2}$$

$$= -\frac{x}{3}(3-2x)^{3/2} - \frac{1}{15}(3-2x)^{5/2}$$