Review Problems for the Final Exam

Calculus B - Fall 2008

For problems 1-10, find the antiderivative.

1. $\int x\, e^{3x^2}\, dx$

2. $\int x\, e^{3x}\, dx$

3. $\int \frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}\, dx$

4. $\int \sqrt{x}\ln(x)\, dx$

5. $\int x^5 \sin(x^3)\, dx$

6. $\int \frac{2x+4}{x^2+2x+4}\, dx$

7. $\int \frac{2x+4}{x^2+5x+4}\, dx$

8. $\int \frac{x^3}{x^2+1}\, dx$

9. $\int \sec^2(x)\tan(x)\, dx$

10. $\int \frac{x}{\sqrt{4-x^2}}\, dx$

For problems 11-13, let

$$I = \int_0^1 e^{-x^2}\, dx.$$

11. Estimate I using a trapezoid sum with 4 subintervals (I want to see all 4 terms in your sum, so do not give a final answer in sigma notation.)

12. Put into sigma notation the right endpoint Riemann sum R_n estimating I.

13. Find an integer n such that $|I - R_n| \leq 0.001$. Make sure you justify your answer.

14. When approximating the integral $J = \int_0^1 e^{x^2}\, dx$ with Riemann sums, arrange the following in order from least to greatest: J, L_{10}, R_{10}, M_{10}, T_{10}.

For problems 15-16, consider the following initial value problem.

$$y' = \frac{x}{y}, \qquad y(0) = 2.$$

15. Use Euler's method with step size 1 to estimate $y(3)$.

16. **(a)** Find the equation for y. Your answer needs to be in the form $y(x) = \ldots$.

(b) Evaluate $y(3)$ exactly.

For problems 17-18, evaluate the improper integral.

17. $\displaystyle\int_1^\infty \frac{1}{x^{\frac{5}{4}}}\,dx$

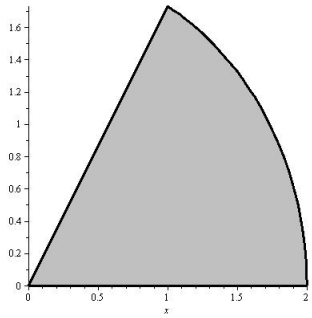
18. $\displaystyle\int_{-\infty}^\infty x^2 e^{-x^3}\,dx$

For problems 19-21, determine whether or not the improper integral converges. If it converges, find a proper integral that approximates the improper integral within 0.01*. Note: You do not need to evaluate either integral.*

19. $\displaystyle\int_1^\infty \frac{dx}{x-\ln(x)}$

20. $\displaystyle\int_{\sqrt{\pi}}^\infty e^{-x}\frac{\sin(x)+\cos(x)}{17}\,dx$

21. $\displaystyle\int_1^\infty \frac{4-2*\cos(\sqrt{x})}{x^2+3\pi}\,dx$



For problems 22-25, let R be the region (shown above) enclosed by the curves

$$y = 0, \quad y = \sqrt{3}\,x, \quad \text{and} \quad y = \sqrt{4 - x^2}.$$

Also, let S be the solid obtained by revolving R around the x-axis.

22. Evaluate the area of R.

23. Evaluate the perimeter of R.

24. Evaluate the volume of S.

25. Find the x-coordinate of the center of mass of S.

26. A certain Jack-in-the-Box is at rest when the head and spring are 20 cm above the box lid. One Joule (Newton-meter) of work is required to compress the spring 20 cm and get Jack back in the box. How much force does the lid need to be able to withhold in order to keep the spring compressed and Jack in the box?

27. After an exciting day of sight seeing in Mount Vernon, you head back to Gambier. Unfortunately, this Ohio winter day has left Wiggin St. covered in ice, and your car is unable to make it up the hill. You walk up to your dorm room, find your 1/4 mile long heavy duty rope, and round up your strongest friends. How much work is required to stand at the top of the hill and pull your $3,000$ pound car 1000 feet up the hill to you? Assume the rope weighs 1 lb/ft. (With the hill being so icy, it is safe to assume that friction will not contribute to your workload.)

28. **(a)** Find the 3^{rd} order Maclaurin polynomial, $M_3(x)$, for $f(x) = \frac{1}{1-2x}$.

(b) Find a bound on the error committed by the approximation $|f(x) - M_3(x)|$ over the interval $[-1/4, 1/4]$.