Review Problems for Exam 2

Calculus B - Fall 2008

For problems 1-4, determine whether or not the improper integral converges. If it converges, evaluate the integral. If it diverges, specify whether the integral diverges to ∞, $-\infty$, or neither.

1. $\displaystyle\int_1^{\infty} \frac{dx}{x^{3/2}}$

2. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{(1+x^2)^2}\,dx$

3. $\displaystyle\int_{-\infty}^{\pi} e^x \sin x\,dx$

4. $\displaystyle\int_1^{e} \frac{dx}{x\,\ln x}$

For problems 5-6, determine whether the improper integral converges or diverges. Justify your answer.

5. $\displaystyle\int_1^\infty \frac{dx}{x^2 + \arctan x}$

6. $\displaystyle\int_{10}^\infty \frac{2x^3 + 7}{(x-3)^4}\,dx$

For problems 7-8, find a proper integral that approximates the improper integral within 0.01*. Note: You do not need to evaluate either integral.*

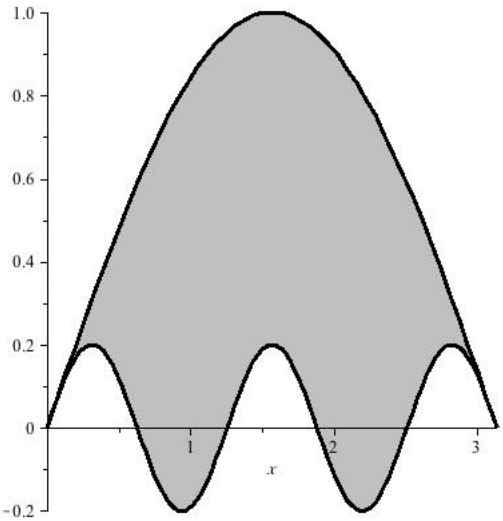
7. $\displaystyle\int_0^\infty \frac{\arctan x}{1 + x^2 + \sqrt{3x}}\,dx$

8. $\displaystyle\int_0^\infty e^{-x}(2 - \sin(1/x))\,dx$

For problems 9-10, let A be a solid whose base is the region bounded by the parabola $y = x^2$ and the lines $y = 0$ and $x = 3$. Cross sections of A perpendicular to the x-axis are squares.

9. Evaluate the volume of A.

10. Find the x-coordinate of the center of mass of A.



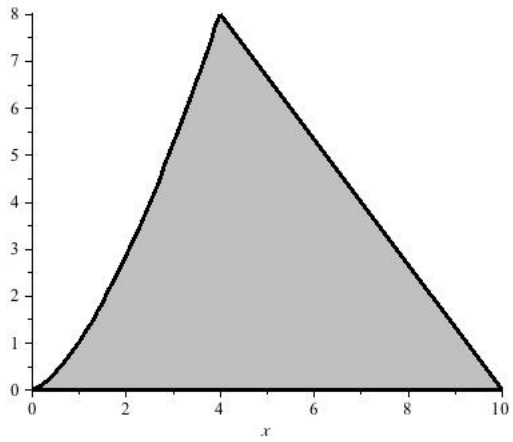
For problems 11-12, let R be the region (shown above) enclosed by the curves

$$y = \sin x \quad \textit{and} \quad y = \frac{\sin(5x)}{5}$$

between $x = 0$ and $x = \pi$.

11. Evaluate the area of R.

12. Find but do not evaluate an expression that calculates the perimeter of R.



For problems 13-15, let S be the region (shown above) enclosed by the curves

$$y = x^{3/2}, \qquad y = \frac{4}{3}(10 - x), \quad \text{and} \quad y = 0.$$

13. Evaluate the area of S.

14. Calculate the perimeter of S.

15. Find the volume of the solid created by rotating S around the y-axis.

16. While studying calculus on a Friday night, you get hungry and order a pizza. Since Taylor's Theorem is too interesting to set down, you pull the pizza up to your 4^{th} floor window using a basket and rope. How much work is required to lift the 12 pound basket 35 feet to your window using a rope weighing 0.2 lb/ft?

17. Suppose 25 N of force is required to hold a spring stretched 4 cm from rest. How much work is required to stretch the same spring from 4 cm out to 10 cm from rest.

For problems 18-20, solve the initial value problem.

18. $\frac{dy}{dt} + ty = y, \quad y(0) = 3.$

19. $x^2y' = y - xy, \quad y(1) = 4.$

20. $\frac{dy}{dx} = \frac{y^3}{x^2}, \quad y(1) = 1.$

21. (a) Find the 4^{th} order Maclaurin polynomial, $M_4(x)$, for $f(x) = \cos(2x)$.

(b) Find an interval I centered at $x = 0$ such that for all x in I,

$$|f(x) - M_4(x)| < 0.1.$$

22. (a) Let P_n be the n^{th} order Taylor polynomial for $g(x) = \ln x$ based at 1. Find an integer n for which $|P_n(2) - \ln 2| < 10^{-5}$.

(b) For the n you found in part (a), write $P_n(x)$ in sigma notation.