Review Problems for Sequences and Series

Calculus B - Fall 2008

For problems 1-5, determine whether or not the given sequence converges. If it converges, evaluate the limit. If it diverges, specify whether the sequence diverges to ∞, $-\infty$, or neither.

1. $\left\{\dfrac{2k^2+\pi}{3k^2-5k+700}\right\}_{k=1}^{\infty}$

2. $\left\{\dfrac{\sin(k^2+3k-1)}{k}\right\}_{k=3}^{\infty}$

3. $\left\{\dfrac{2^k}{3+2^k}\right\}_{k=0}^{\infty}$

4. $\left\{\dfrac{k^3(2k)!}{(2k+2)!}\right\}_{k=1}^{\infty}$

5. $\left\{\dfrac{(-5)^k}{k^{20}}\right\}_{k=10}^{\infty}$

For problems 6-10, write the geometric series in the form $\sum_{k=0}^{\infty} ar^k$. Then calculate the sum.

6. $1 + \frac{1}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi^3} + \frac{1}{\pi^4} + \cdots$

7. $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \frac{1}{243} - \cdots$

8. $\sum_{k=n+1}^{\infty} \frac{4}{5^k}$

9. $\frac{7}{16} + \frac{7}{32} + \frac{7}{64} + \frac{7}{128} + \cdots$

10. $\sum_{k=0}^{\infty} \frac{2^{3k+1}}{3^{2k-1}}$

For problems 11-15, determine whether or not the series converges. If it converges, evaluate the sum. If it diverges, specify whether the series diverges to ∞, $-\infty$, or neither.

11. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2+5k+6}$

12. $\displaystyle\sum_{k=1}^{\infty} \ln\left(\sqrt[k]{2}\right)$

13. $\displaystyle\sum_{k=0}^{\infty} \frac{2^{k+1}+8}{3^k}$

14. $4+3+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$

15. $\displaystyle\sum_{k=4}^{\infty} \frac{2^k}{k^3}$

For problems 16-18, show that the series converges. Then find a partial sum that estimates the sum within 0.001*. Note: You do not need to evaluate the partial sum.*

16. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$

17. $\displaystyle\sum_{k=1}^{\infty} \frac{\sin(2k)}{k^2}$

18. $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k+1}}{3^k + 8}$

For problems 19-22, find the interval of convergence for the given power series.

19. $\displaystyle\sum_{k=1}^{\infty} \frac{3^k x^k}{(k+1)^2}$

20. $\displaystyle\sum_{n=1}^{\infty} n!(2x-1)^n$

21. $\displaystyle\sum_{j=1}^{\infty} \frac{(-2)^j}{\sqrt{j}}(x-3)^j$

22. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{x^n}{4^n \ln(n)}$

For problems 23-26, find the Maclaurin series of the function and give the radius of convergence.

23. $p(x) = \dfrac{1}{1-2x^2}$

24. $q(x) = \dfrac{\sin(x)}{x}$

25. $f(x) = e^{-x^2}$

26. $g(x) = \displaystyle\int e^{-x^2}\,dx$

27. Using a power series, estimate $\int_0^1 e^{-x^2}\,dx$ within 0.01.

28. Let $p(x)$ be as defined in Problem 23. Find $p^{(35)}(0)$.

29. Let $f(x)$ be as defined in Problem 25. Find $f^{(50)}(0)$.

30. Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}.$$