Sections 2.4 and 2.5: Infinite Limits, Limits at Infinity and Evaluating Limits

To evaluate limits it is important to analyze what each function is doing as $x \to a$ or $x \to \pm\infty$ before doing any computation. It is also useful to know the behavior of basic functions of Calculus by heart. This includes knowing their end behavior. By end behavior of a function f we mean $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Here are the end behaviors of several functions.

$\lim_{x\to\infty} e^x = \infty$	$\lim_{x\to-\infty} e^x = 0$	$\lim_{x\to\infty} e^{-x} = 0$	$\lim_{x\to-\infty} e^{-x} = \infty$	$\lim_{x\to\infty} \ln(x) = \infty$	$\lim_{x\to0^+} \ln(x) = -\infty$
$\lim_{x\to\pm\infty} \sin(x) = DNA$	$\lim_{x\to\pm\infty} \cos(x) = DNA$	$\lim_{x\to\infty} \arctan(x) = \frac{\pi}{2}$	$\lim_{x\to-\infty} \arctan(x) = -\frac{\pi}{2}$		

For rational functions, that is functions of the form $\frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials, the end behavior depends on the comparison of degrees of $p(x)$ and $q(x)$. See Theorem 2.7 in the textbook.

Although it is not about the end behavior, a very important limit in Calculus is $\lim_{x\to0} \frac{\sin(x)}{x} = 1$.

E1: a) $\lim_{x\to3} \frac{x^2-3}{(x^2-9)^{2020}}$ b) $\lim_{x\to0^+} \frac{e^x}{x}$ c) $\lim_{x\to0^-} \frac{e^x}{x}$ d) $\lim_{x\to0} \frac{e^x}{x}$

E2: a) $\lim_{x\to0} \frac{e^x}{x^2}$ b) $\lim_{x\to1} \frac{e^x}{\ln(x)}$

E3: $\lim_{x\to0^+} \frac{e^x}{\ln(x)}$ b) $\lim_{x\to0} e^{\frac{-1}{x^2}}$

E4: a) $\lim_{x\to-\infty} \frac{x^{10}+1}{2x^{10}-x-3000}$ b) $\lim_{x\to\infty} \frac{2\cdot x^5+10^6}{x^6-500}$ c) $\lim_{x\to\infty} \frac{2x^3+6x-1}{x^2+2}$

E5: a) $\lim_{x\to-\infty} e^{-x}\cdot\ln(-x)$ b) $\lim_{x\to\infty}\cos(\frac{1}{x})$ c) $\lim_{x\to\infty}\frac{\cos(x)}{\ln(x)}$

E6: a) $\lim_{x\to\infty} x\cdot\sin(\frac{1}{x})$ b) $\lim_{x\to-\infty} x\cdot\sin(\frac{2}{x})$

E7: a) $\lim_{x\to\infty}\left(\sqrt{x^2+3\cdot x}-x\right)$ b) $\lim_{x\to-\infty}\left(\sqrt{x^2+3\cdot x}-x\right)$

E8: $\lim_{x\to-\infty}\left(\sqrt{x^2+2x}+x\right)$