Homework 1, Due Monday, Jan 22

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Define the complex conjugation $\phi : \mathbb{C} \to \mathbb{C}$ by $\phi(a + bi) = \overline{a + bi} = a - bi$. Is complex conjugation a ring homomorphism? Is it also a ring isomorphism?
2. Let R be a ring such that $(R, +)$ is cyclic. Show that R is commutative.
3. Let $(A, +)$ be an abelian group and let $End(A)$ be the set of all endomorphisms of A, i.e. homomorphisms of A to itself. Define addition and multiplication on $End(A)$ by $(f + g)(a) = f(a) + g(a)$ and $(f \circ g)(a) = f(g(a))$. Show that $(End(A), +, \cdot)$ is a ring.
4. Let R be a ring, and X be a non-empty set. Let $R^X := \{f : X \to R\}$ be the set of all functions from X to R. Define $+$ and $\cdot$ on R^X as follows: $(f + g)(x) = f(x) + g(x)$ and $(f \cdot g)(x) = f(x) \cdot g(x)$. Show that R^X is a ring with these operations. Is this ring commutative?