

Definitions of the Functions for Lab 9

When you use a calculator or computer to compute the value of transcendental function such as $e^{2.5}$, they typically use a partial sum of the power series expansion for that function. Note that the answers you get are approximations (often the exact values are irrational, we get rational approximations to those exact values). By taking more terms in the expansion, we can improve the precision of our approximation. In this lab, we will take first few initial terms of the power series expansions of the functions e^x (typically called $\exp(x)$ in computer algebra systems), $\sin(x)$, $\cos(x)$ and $\log(x)$ (for $\ln(x)$). Use the following formulas in your implementation, exactly as given. [Note: \dots means to get the exact value, one needs to compute the infinite sum. We are getting an approximation by truncating the sum at a point.]

$$e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \text{ for all real numbers } x$$

$$\sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \text{ for all real numbers } x$$

$$\cos(x) \approx 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \text{ for all real numbers } x$$

$$\ln(x) \approx (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} - \frac{(x-1)^6}{6} + \dots \text{ for } 0 < x < 2.$$