
Homework 9, Due Monday, Dec 8

This homework must be done individually. Remember to follow Math department's guidelines for homework, and the course policy on the use of AI tools. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Let a group G act on a set X and let $x, y \in X$ be such that $y = g \cdot x$ for some $g \in G$.
 - (a) Show that $gG_xg^{-1} \subseteq G_y$ (recall: G_x denotes the stabilizer of x).
 - (b) Explain why it follows from part (a) that $gG_xg^{-1} = G_y$
 - (c) Show that if G acts transitively on X then the kernel of the action is $\bigcap_{g \in G} gG_xg^{-1}$
2. Let A be a set and let $G \leq S_A$. Let $\sigma \in G$, and $a \in A$.
 - (a) Show that $\sigma G_a \sigma^{-1} = G_{\sigma(a)}$
 - (b) Show that if G acts transitively on A , then $\bigcap_{\sigma \in G} \sigma G_a \sigma^{-1} = \iota$, where ι is the identity map on A .
3. Find the number of distinguishable ways in which the edges of a square can be painted if 6 different colors of paint are available and
 - (a) no color is used more than once
 - (b) the same color can be used on any number of edges
4. Let A, B be groups with $C \trianglelefteq A$ and $D \trianglelefteq B$. Show that
 - (a) $(C \times D) \trianglelefteq (A \times B)$
 - (b) $(A \times B)/(C \times D) \cong (A/C) \times (B/D)$