
Homework 8, Due Monday, Nov 10

This homework must be done individually. Remember to follow Math department's guidelines for homework, and the course policy on the use of AI tools. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Prove that $A_n \trianglelefteq S_n$. What familiar group is S_n/A_n isomorphic to?
2. Let K be a subgroup of $\langle \mathbb{R}^*, \cdot \rangle$, and let $H = \{A \in GL(2, \mathbb{R}) : \det(A) \in K\}$. Show that $H \trianglelefteq GL(2, \mathbb{R})$.
3. Let $H \trianglelefteq G$ where G is a finite group. Show that the order of an element gH in G/H must divide the order of g in G .
4. Let $H \trianglelefteq G$, $g \in G$ where G is a finite group. If gH has order 4 in G/H and $|H| = 12$, what are the possibilities for $|g|$ in G ?
5. An element y is called a square in a group G if $y = x^2$ for some $x \in G$. Suppose G is an abelian group and $H \trianglelefteq G$. If every element of H is a square, and every element of G/H is a square, prove that every element of G is a square.