Homework 6, Due Monday, Oct 27

This homework must be done individually. Remember to follow Math department's guidelines for homework, and the course policy on the use of AI tools. . Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Let U be the subgroup of $\langle \mathbb{C}^*, \cdot \rangle$ consisting of all complex numbers of modulus (absolute value) 1. Give a geometric description of the cosets of U.

2. Suppose that a group G contains elements of orders 1,2,3,4,5,6,7. What is the smallest possible order of G?

3. Suppose H and K are subgroups of a group G. Given that $|H| = 24$ and $|K| = 35$, determine $H \cap K$.

4. Suppose G is a finite group of order p^2, where p is a prime number. Show that either G is cyclic, or $g^p = e$ for all $g \in G$.

5. Let G be a finite group and p be a prime number. Assume that G contains some elements of order p. Show that the number of elements of order p in G is a multiple of $p - 1$.