

# Foundations, Gift 11

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Due Friday, December 13

This homework must be done individually. Remember to follow Math department's guidelines for collaboration on homework. Please write your solutions independently and neatly. Typesetting in L<sup>A</sup>T<sub>E</sub>X is highly appreciated and encouraged.

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1. Let  $A, B, C, D$  be sets such that  $\text{card}(A) = \text{card}(C)$  and  $\text{card}(B) = \text{card}(D)$ . Prove the following statements
  - (a) If  $A$  and  $B$  are disjoint, and  $C$  and  $D$  are disjoint, then  $\text{card}(A \cup B) = \text{card}(C \cup D)$ .
  - (b)  $\text{card}(A \times B) = \text{card}(C \times D)$
2. Show that the set  $\mathbb{Q}[x]$  of all polynomials with rational coefficients is denumerable. Recall that a polynomial is an expression of the form  $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$  for some integer  $n \geq 0$ . To say that  $p(x)$  is a polynomial over  $\mathbb{Q}$  means that  $a_i \in \mathbb{Q}$  for all  $0 \leq i \leq n$ .