Islamic Constructions: The Geometry Needed by Craftsmen

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Abstract

The Islamic world has a rich artistic tradition of creating highly geometric and symmetric ornamentation. Over the centuries, the process of creating Islamic tilings was refined from the 15th Century ornamentation in the Alhambra Palace in Granada, Spain to the exquisite tilings, which are seen in mosques, mausoleums and minarets throughout the world today. The contemporary mathematics of group theory and knot theory combined with computer programs provide tools for creating modern day variations of these historical tilings.

The title of this paper is motivated by the 10th Century treatise On Those Parts of Geometry Needed by Craftsmen written by the Khorasan mathematician and astronomer Abu'l-Wafã who described several constructions made with the aid of straightedge and "rusty compass", a compass with a fixed angle. He was one of a long line of Islamic mathematicians who developed geometric techniques that proved useful to artisans in creating the highly symmetrical ornamentation found in architecture around the world today. This paper looks at the geometry of Abu'l-Wafã with an eye toward determining geometric methods for reproducing Islamic tilings with students in the classroom.

1. Islamic Ornamentation

The Islamic world has a rich heritage of incorporating geometry in the construction of intricate designs that appear on architecture, tile walkways as well as patterns on fabric. This highly stylized form of art has evolved over the centuries from simple designs to fairly complex geometry involving a high degree of mathematical symmetry (Figure 1). The Alhambra Palace, the 15th Century Moorish architectural wonder, in Granada, Spain contains many excellent examples of these Islamic constructions used to ornament this 15th Century Moorish architectural wonder.

Figure 1: *Islamic Tiling from the Alhambra*

The interplay between mathematics and art involved in constructing these patterns has a rich history of liaisons between mathematicians and artisans. These collaborations point out the similarities and differences in the ways in which these creative individuals approach their crafts. On the one hand, geometers use proof and rigor to verify that constructions are exact while; artisans use constructions, which are not exact to appeal to a sense of aesthetic. Consider a simple construction of an isosceles right

triangle with legs each one unit in length. The geometer would appeal to the Pythagorean theorem to conclude that the hypotenuse has a length of $\sqrt{2}$. The artisan may be completely unaware of this fact and further, the very act of constructing such a triangle in clay, wood, or some other material makes such accuracy of little importance.
This interdisciplinary relationship between mathematics and art also serves as a powerful tool for teachers in the classroom. These elaborate visual designs provide motivating examples for exciting students to the wonderful artistic applications of symmetry and geometry. It is with this backdrop that we focus on a historical example of such a collaboration followed by construction examples for the classroom.

2. Historical Example of Collaboration

The history of geometric design contains many instances of collaborations between mathematicians and artists and the continuing evolution of Islamic ornamentation is no exception. During the 10th Century, the Islamic mathematician and astronomer Abu'l-Wafā, participated in meetings in Baghdad [1], which served as a forum for artisans and mathematician to discuss methods for constructing ornamental designs in wood, tile, and other materials.

In his treatise, *On Those Parts of Geometry Needed by Craftsmen*, Abu'l-Wafā discussed mistakes that craftsmen make in constructing geometric designs. These mistakes point out the differences between the aesthetic considerations of the artisan and the precise calculations of the mathematician. In chapter, *On Assembling and Dividing Squares*, Abu'l-Wafā describes the techniques by which artisans mask slight imperfections in their constructions. In fact, the tools of the artisan, no matter how fine usually will contain a certain degree of approximation when used to construct a design.

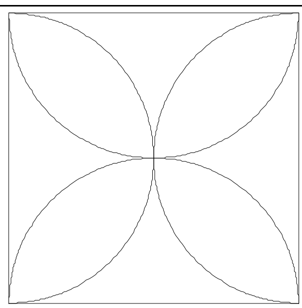

Figure 2: *Semicircles*

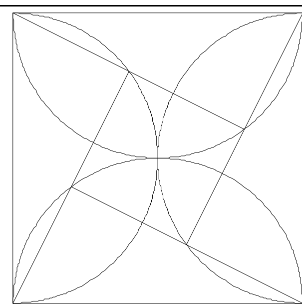

Figure 3: *Smaller Square*

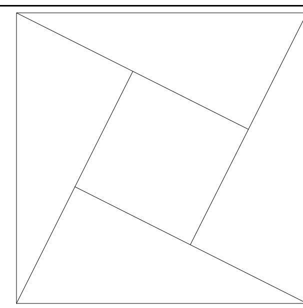

Figure 4: *Assembled Squares*

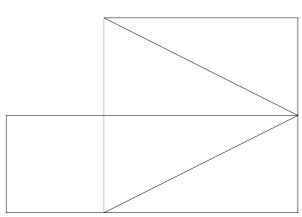

Figure 5: *Divided as Two Squares*

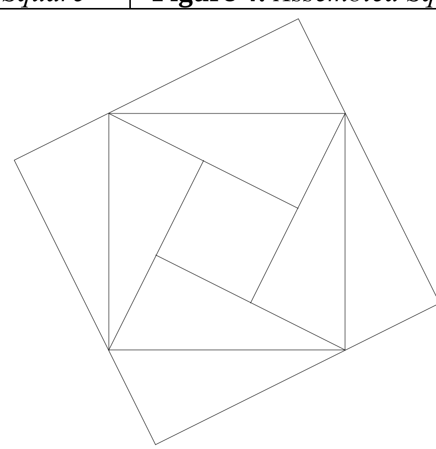

Figure 6: *Additional Square*

Abu'l-Wafā described several constructions with a straightedge and "rusty compass" [2] having one fixed radius. These included constructing a perpendicular at the endpoint of a line segment, dividing segments in equal parts, bisecting angles, constructing a square in a circle and, constructing a regular pentagon. These constructions form the basis for creating many of the symmetric patterns of the artisans at that time.

In one example, a method for dividing a square begins by constructing semicircles on the four sides of a square (Figure 2). A point is chosen on each semicircle and a line is drawn from the point to a vertex of the square (Figure 3). Since any triangle inscribed in a semicircle must be a right triangle, it follows that the central polygon is a square regardless of where the initial point is chosen on the semicircle. In this figure, the point is chosen so that the longer leg of the right triangle is twice as long as the shorter leg. Removing the semicircles (Figure 4) reveals a square, which may be divided into two smaller squares (Figure 5). Adding four right triangles produces a larger square (Figure 6). Once this square tile was constructed, the artisan could extend the pattern in two perpendicular directions (Figure 7).

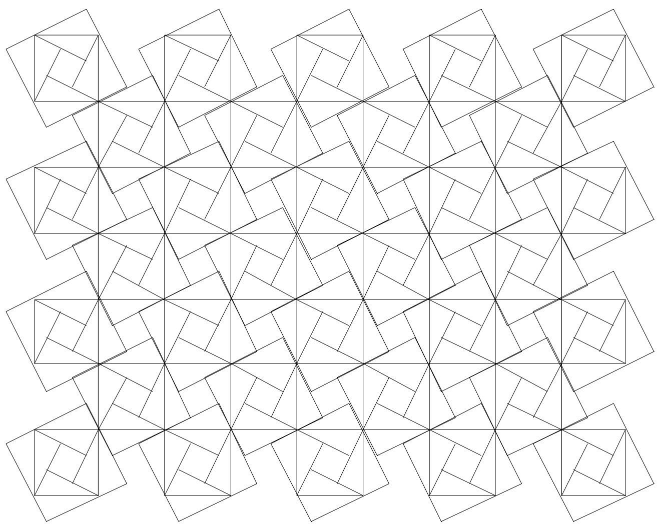

Figure 7: *Tiling by Translations*

In subsequent centuries, many other Islamic mathematicians sought to apply geometry to solving problems that were important to artisans. In the 11^{th} Century, Omar Khayyam used a cubic equation to construct a right triangle whose hypotenuse is equal to the sum of the short side and the perpendicular to the hypotenuse.

3. Construction Examples for the Classroom

A useful tool in teaching mathematics lies in describing cultural connections to the mathematics that is being discussed. This is particularly powerful if the connection being made is to the student's own personal history and culture. For students in the Middle East, these historical references of mathematicians interacting with artisans serve as a springboard for developing the more theoretical ideas of constructions, transformation geometry, and group theory.

In looking at any particular Islamic tiling, it is interesting to investigate methods for constructing it with straightedge and compass or by a computer program like Geometer's Sketchpad or Mathematica. Here is a construction for the fundamental tile that generates the tiling shown in Figure 8. It involves only basic constructions of the type described by Abu'l-Wafā in his treatise. First, concentric squares are drawn (Figure 9) and a circle is constructed in the center of the squares (Figure 10). A square is inscribed in the smaller circle and the lines extended to the large square (Figure 11). Lines are drawn connecting vertices of the inscribed square to midpoints of the large square (Figures 12 and 13). The circle is removed (Figure 14). The lines are widened to form the latticework (Figure 15). All intersecting lines are eliminated from the tile (Figure 16). The overlapping detail of the tiles is created by alternating crossings of the latticework between over and under Figure 17).

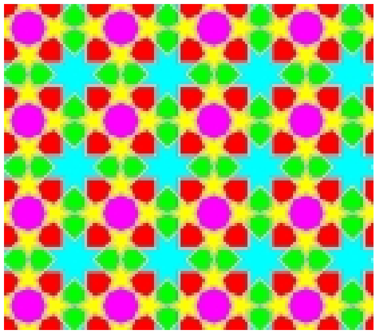

Figure 8: *Islamic Pattern*

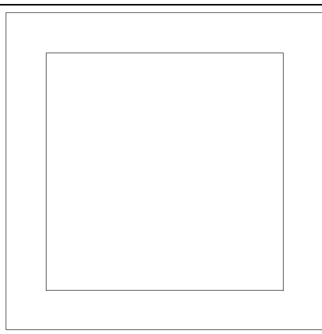

Figure 9: *Centered Squares*

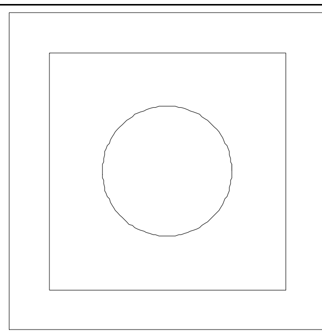

Figure 10: *Centered Circle*

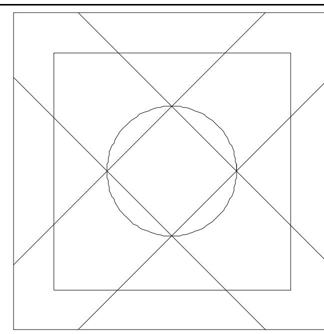

Figure 11: *Perpendiculars*

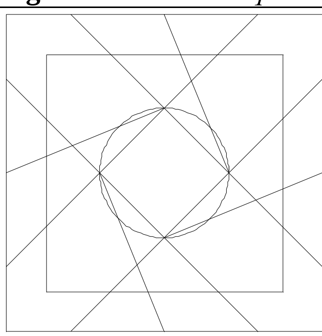

Figure 12: *Spokes*

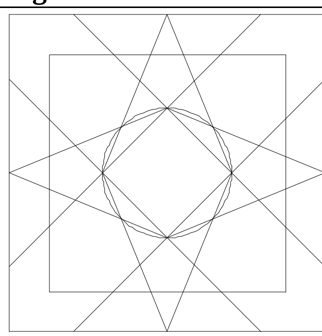

Figure 13: *Star Pattern*

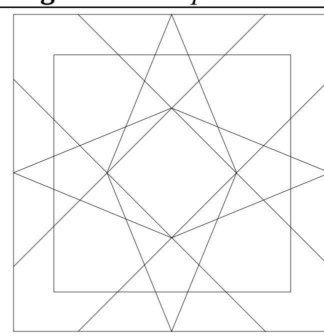

Figure 14: *Circle Removed*

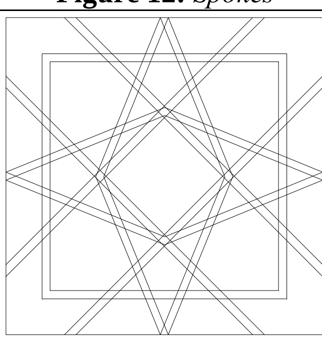

Figure 15: *Paths Drawn*

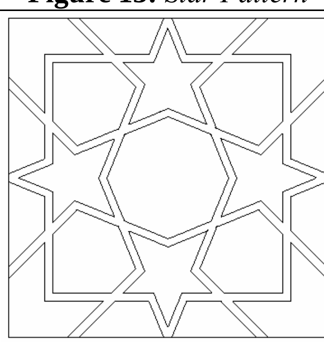

Figure 16: *Lattice Refined*

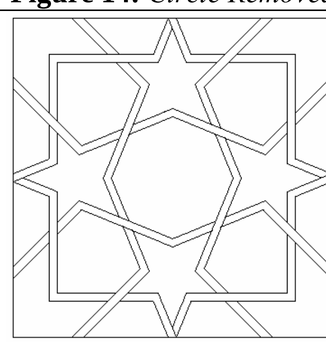

Figure 17: *Overlapping*

Another approach to constructing the tiling is to determine the smallest part of this tile, which could be transformed around so as to create the entire tiling. For this particular tile (Figure 18), the isosceles triangle (Figure 19) would generate the tile by 90-degree rotations. If the overlapping of the latticework were eliminated, the tile could be generated by the smaller triangle (Figure 20).

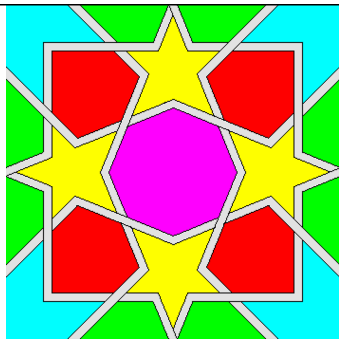

Figure 18: Color Tiling

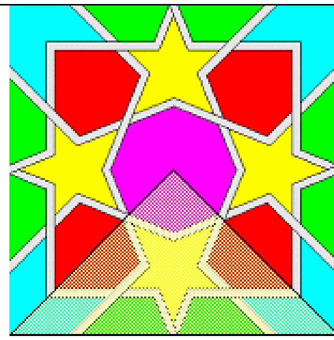

Figure 19: *Fundamental Tile*

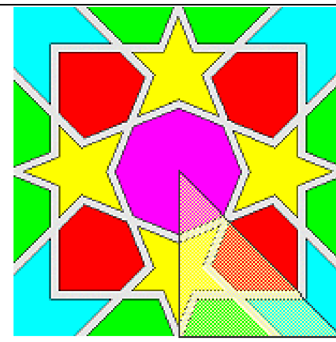

Figure 20: *Fundamental Tile*

4. Conclusion

The construction of Islamic tilings lies on the interesting boundary between mathematics and art. These constructions have a rich history involving both mathematicians and artisans and this history provides motivation to increase student interest and excitement in mathematics.

References

[1[Alpay Ozdural, *Mathematics and Art: Connections between Theory and Practice in the Medieval Islamic World*, Historia Mathematica, Vol. 27, pp. 171-201, 2000.

[2] J.L. Berggren, *Episodes in the Mathematics of Medieval Islam*, Springer-Verlag, 1986.