
Homework 9, Due Monday, Dec 9

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

1. Let a group G act on a set X and let $x, y \in X$ be such that $y = g \cdot x$ for some $g \in G$.
 - (a) Show that $gG_xg^{-1} \subseteq G_y$ (recall: G_x denotes the stabilizer of x).
 - (b) Explain why it follows from part (a) that $gG_xg^{-1} = G_y$
 - (c) Show that if G acts transitively on X then the kernel of the action is $\bigcap_{g \in G} gG_xg^{-1}$

2. Let A be a set and let $G \leq S_A$. Let $\sigma \in G$, and $a \in A$.
 - (a) Show that $\sigma G_a \sigma^{-1} = G_{\sigma(a)}$
 - (b) Show that if G acts transitively on A , then $\bigcap_{\sigma \in G} \sigma G_a \sigma^{-1} = \iota$, where ι is the identity map on A .

3. Find the number of distinguishable ways in which the edges of a square can be painted if 6 different colors of paint are available and
 - (a) no color is used more than once
 - (b) the same color can be used on any number of edges

4. Let A, B be groups with $C \trianglelefteq A$ and $D \trianglelefteq B$. Show that
 - (a) $(C \times D) \trianglelefteq (A \times B)$
 - (b) $(A \times B)/(C \times D) \cong (A/C) \times (B/D)$