**Homework 7, Due Monday, Nov 4**

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

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1. Let $G = \langle \mathbb{C}^*, \cdot \rangle$. Show that the map $\phi : G \to G$ defined by $\phi(x) = x^4$ is a homomorphism and determine its kernel. Determine the kernel of the map $\phi(x) = x^n$ for any positive integer $n$.

2. For a pair of positive integer $m$ and $n$ define $\phi : \mathbb{Z} \to \mathbb{Z}_m \times \mathbb{Z}_n$ by $x \to (x \mod m, x \mod n)$. Show that this map is a homomorphism and find its kernel. Express the kernel as a familiar subgroup of $\mathbb{Z}$. What is the kernel in the case $\gcd(m, n) = 1$?

3. Let $\phi : G \to G'$ be a group homomorphism where $|G|$ is a prime. Show that $\phi$ must be either the trivial homomorphism or must be one-to-one.

4. Let $\phi$ be a homomorphism from $\mathbb{Z}_{40}^*$ to $\mathbb{Z}_{40}^*$ with $\ker(\phi) = \{1, 9, 17, 33\}$. If $\phi(3) = 11$, find all elements of $\mathbb{Z}_{40}^*$ that map to 11.

5. Let $G$ be a group and let $h, k \in G$. Define $\phi : \mathbb{Z} \times \mathbb{Z} \to G$ by $\phi(m, n) = h^m k^n$. Find a necessary and sufficient condition for $\phi$ to be a homomorphism for all choices of $h$ and $k$ in $G$. Of course, justify your answer.